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ABSTRACTS

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Generalized Orthogonal Polynomials And Associated Semigroups.

Cristina Balderrama

Abstract

We construct and study orthogonal basis of generalized polynomials associated to a real finite measure μ. A generalized polynomial is a central function, that is to say, a function with argument in the space of hermitian matrices H_n that only depends on the eigenvalues of the of the hermitian matrix in which it is being evaluated. For their con- struction we begin with the family of orthogonal polynomials on $\mathbb{R}$ with respect to the measure μ and construct a family of related symmetric orthogonal polynomials on $\mathbb{R}^n$ to then use a bijection between the space of symmetric polynomials on Rn and the space of central functions on H_n. For a given family of orthogonal polynomials on $\mathbb{R}$ it is possible to con- struct a Markov semigroup of operators having them as eigenfunctions using the corresponding Markov generator sequence. From the Markov semigroup associated to the orthogonal polynomials with respect to the measure μ on $\mathbb{R}$, we construct a semigroup of operators with eigenfunctions given by the related generalized polynomials. We characterize the infinitesimal generator of this semigroup and we give necessary conditions for it to be Markov.

Some discrete problems in harmonic analysis

Anthony Carbery.

Abstract

We discuss three discrete problems in harmonic analysis which are linked in two distinct ways.

Mappings of Lie groups that preserve cosets and isomorphisms of the Fourier–Stieltjes algebras

Michael Cowling

Abstract

This talk is about bijections $\phi : G_1 \to G_2$ of connected Lie groups. The aim is to prove the following results:

1. if ϕ sends cosets of subgroups of G_1 onto cosets of subgroups of G_2, then ϕ is composed of one or more of the following: a group isomorphism, a translation, and inversion.

2. if the mapping $u \mapsto u \circ \phi$ is an isomorphism of $B(G_2)$ onto $B(G_1)$, then ϕ is composed of one or more of the following: a group isomorphism, a translation, and inversion.

Curiously enough, for semisimple groups, things are easy, while for nilpotent groups, things are much harder.

Uncertainty Principles For Certain Integrals Transforms

Radouan Daher

Abstract

In this talk we shall discuss generalizations of three uncertainty principlles, Miyachi's theorem, Beurling's theorem, and Morgan's theorem for certains integrals transforms. More precisely we give analogues of theses theorems for the Cherednick transform.

(Recent joint work with S.L. Hamad, T. Kawazoe, and T. Shimeno)

Local L^p-L^q boundedness of Bergman projections in tubes over cones

Gustavo Garrigos

Abstract

In this talk we present work in progress concerning local L^p-L^q bounds for Bergman projections in tubes over cones. Via conformal mappings, this property implies the L^p-L^q boundedness of the (global) Bergman projection in the associated bounded symmetric domains. In particular, we obtain optimal L^∞-L^q bounds (in cones of arbitrary rank) for the operator with positive kernel, and by interpolation, new results of type L^p-L^q for the usual projection.

This is joint work with A. Bonami and C. Nana.

The multivariable strong maximal function

Loukas Grafakos

Abstract

A multivariable version of the strong maximal function is introduced and a sharp distributional estimate for this operator in the spirit of the Jessen, Marcinkiewicz, and Zygmund theorem is obtained. Conditions that characterize the boundedness of this operator on products of weighted Lebesgue spaces equipped with multiple weights are obtained.

Harmonic analysis of the Boltzmann collision operator

Philip Gressman

Abstract

This topic will give a brief introduction to the bilinear Boltzmann collision operator. This operator may be viewed as a singular, bilinear, Radon-like operator with many features that are interesting from a harmonic analysis standpoint, independently of its physical significance. Specifically I will discuss recent sharp estimates of the collision operator in weighted, non-isotropic L^2-Sobolev spaces. This is the product of joint work with R.M. Strain.

Refinements of the Hardy and Morgan uncertainty principles

Takeshi Kawazoe

Abstract

Various generalizations of Hardy's theorem and Morgan's theorem, which assert that a function on $\mathbb{R}$ and its Fourier transform cannot both be very small, are known. We give two theorems which improve various generalizations known so far. This is a joint work with J. Liu and A. Miyachi.

Ideal structure of operator space tensor product of C^*-algebras

Ajay Kumar

Abstract

For C^*-algebras A and B and $p, q \in \mathbb{N}$, let $V = (v_{ij}) \in \mathbb{M}_p(A)$ and $W = (w_{ij}) \in \mathbb{M}_p(B)$, define the tensor product $V \otimes W$ to be the $pq \times pq$ matrix

$$V \otimes W = (v_{ij} \otimes w_{kl})$$

in $\mathbb{M}_{pq}(A \otimes B)$ using $(\mathbb{M}_p \otimes A) \otimes (\mathbb{M}_q \otimes B) \simeq \mathbb{M}_{pq}(A \otimes B)$. Each element u in $A \otimes B$ can be written $u = \alpha(V \otimes W)\beta$ for some $\alpha \in \mathbb{M}_{1,pq}, V \in \mathbb{M}_p(A), W \in \mathbb{M}_q(B)$ and $\beta \in \mathbb{M}_{pq,1}$, where $\mathbb{M}_{kl}$ is the space of complex $k \times l$ matrices. The operator space projective norm $\|u\|_\wedge$ is defined on the algebraic tensor $A \otimes B$ by

$$\|u\|_\wedge = \inf\{\|v\|\|V\|\|W\|\|\beta\| : u = \alpha(V \otimes W)\beta \text{ is a decomposition as above}\}.$$

We describe $A \overset{\wedge}{\otimes} B$ as a Banach $*$–algebra and its ideal structure and determines the lattice of closed ideals of $B(H) \overset{\wedge}{\otimes} B(H)$ completely.

The free and Hermite Schrödinger equations: an equivalence

Sanghyuk Lee

Abstract

We consider the Schrödinger equation for the harmonic oscillator, $i\partial_t u = \mathcal{H}u$, where $\mathcal{H} = -\Delta + |x|^2$, with initial data in $H^s(R^d)$. We prove an equivalence between time-space estimates for the harmonic oscillator and those for the free Schrödinger equation. As corollary, we obtain an equivalence for almost everywhere convergence of the solutions to their initial data except the endpoint cases. Consequently, with $d = 2$ we get almost everywhere convergence for $s > 3/8$, improving a result of Yajima who proved the convergence for $s > 1/2$.

This is a joint work with Keith Rogers.

Bi-linear Littlewood-Paley for circle group

Parasar Mohanty

Abstract

In this talk we will see bi-linear analogue of Rubio de Francia's result on Littlewood-Paley square function for circle group $\mathbb{T}$.

Problems of harmonic analysis related to finite type hypersurfaces in $\mathbb{R}^3$

Detlef Müller

Abstract

Consider a smooth hypersurface S in $\mathbb{R}^3$ of finite type, in the sense that every tangent plane has finite order of contact with S, and let $d\mu = \rho d\sigma$ be a surface carried measure with smooth, compactly supported density $\rho \geq 0$ with respect to the surface measure $d\sigma$. In the talk, I shall address three, somewhat related problems associated to this setting:

A. Find the best possible uniform decay estimates for the Fourier transform of the surface carried measure $d\sigma$.

B. If we denote by A_t the averaging operator $A_t f(x) := \int_S f(x-ty)d\mu(y)$, determine for which p's the associated maximal operator

$$\mathcal{M}f(x) := \sup_{t>0} |A_t f(x)|$$

is bounded on $L^p(\mathbb{R}^3)$.

C. Determine the range of exponents p for which a Fourier restriction estimate

$$\left(\int_S |\hat{f}(x)|^2 d\mu(x) \right)^{1/2} \leq C \|f\|_{L^p(\mathbb{R}^3)}$$

holds true. The first problem is classical, and the other ones originate from seminal work by E.M. Stein. In joint work with I. Ikromov, and in parts with M. Kempe, we have given essentially complete answers to these problems (for question B at least if $p > 2$) recently. What may come as a bit of a surprise is that problem A and problem C turn out to be less directly related as one might expect in view of earlier work on Fourier restriction.

Support theorems on $\mathbb{R}^n$ and noncompact symmetric space

E. K. Narayanan

Abstract

We consider convolution equations of the form $f * T = g$ where f and g are in $L^p(\mathbb{R}^n)$, T a compactly supported distribution. Under natural assumptions on the zero set of the Fourier transform of T we show that f is compactly supported, provided g is. Similar results on noncompact symmetric spaces will also be described.

This is joint work with Amit Samanta.

On Weighted $A_p(G)$-Modules

Serap Öztop

Abstract

Let G be a locally compact abelian group with Haar measure, w be a weight function on G and $1 < p < \infty$. The work of R. Spector on $A_p(G)$ has motivated us to be interested in the structure theory of weighted $A_p(G)$ denoted by $A_{p,w}(G)$. We show that $A_{p,w}(G)$ is a Banach $A_p(G)$-module under pointwise multiplication and, as such, a fixed $v \in A_{p,w}(G)$, induces by multiplication an operator T_v from $A_p(G)$ into $A_{p,w}(G)$ defined by $T_v(u) = uv$. Following the work of Friedberg we show that the compact multiplier T_v is trivial if G is nondiscrete. We also study some multiplier problems from $A_p(G)$ to $A_{p,w}(G)$ spaces.

Analyticity of the Schrödinger Propagator on the Heisenberg group

Sanjay Parui

Abstract

We consider the Schrödinger equation associated to the Hermite and special Hermite operators and also to the sublaplacian on the Heisenberg group. The Hermite and Special Hermite cases are straight forward, whereas the sublaplacian case is very subtle. We obtain an isometric isomorphism between certain subspaces of $L^2(H^n)$ and a direct integral of weighted Bergmann spaces related to special Hermite operator.
A joint work with P.K. Ratnakumar and S. Thangavelu.

Semisimple orbital integrals on the symplectic space for a real reductive dual pair

Angela Pasquale

Abstract

Let W be a symplectic real vector space, $Sp(W)$ the corresponding symplectic groups and $\widetilde{Sp}(W)$ the metaplectic group. Let $G, G' \subset Sp(W)$ be a real reductive pair, and let $\widetilde{G}$ and $\widetilde{G'}$ be the preimages of G and G' in $\widetilde{Sp}(W)$, respectively. Furthermore, let $\Pi \otimes \Pi'$ be an irreducible admissible representation of $\widetilde{G} \times \widetilde{G'}$ in Howe's correspondence. We study the regularity properties of the intertwining distribution of $\Pi \otimes \Pi'$ by means of orbital integrals on the odd part of ordinary classical Lie superalgebras.

This is a joint work with M. Mckee and T. Przebinda.

Weak spectral synthesis and weak operator synthesis

R. Prakash

Abstract

Let $\mathcal{A}$ be a commutative, semisimple, regular Banach algebra. Let $\mathcal{A}^*$ be its Banach space dual. Let X be an $\mathcal{A}$- submodule of $\mathcal{A}^*$. Let G be a compact group. We will define the concept of weak X-spectral set for $\mathcal{A}$ and find the relations between weak X-synthesis in the Fourier Algebras $A(G)$ and $A(G \times G)$ and the Varopoulos algebra $V(G)$. The concept of operator synthesis was introduced by Arveson. We will extend recent investigations on operator synthesis by defining and studying, for a $V^{\infty}(G)$-submodule $\mathcal{M}$ of $B(L^2(G))$, sets of weak $\mathcal{M}$-operator synthesis. Relations between X-Ditkin sets and $\mathcal{M}$-operator Ditkin sets and between weak X-spectral synthesis and weak $\mathcal{M}$-operator synthesis will be explored.

Maximal operators and differentiation theorems in sparse sets

Malabika Pramanik.

Abstract

We study maximal averages associated with singular measures on $\mathbb{R}$. Our main result is a construction of singular Cantor-type measures supported on sets of fractional Hausdorff dimension for which the corresponding maximal operators are bounded on L^p for p near 1. As a consequence, we are able to answer a question of Aversa and Preiss on density and differentiation theorems in one dimension. Our proof combines probabilistic techniques with the methods developed in multidimensional Euclidean harmonic analysis, in particular there are strong similarities to Bourgain's proof of the circular maximal theorem in two dimensions. This is joint work with Izabella Laba.

Markov-Bernstein type inequalities and certain class of Sobolev polynomials

Yamilet Quintana

Abstract

Let (μ_0, μ_1) be a vector of non-negative measures on the real line, with μ_0 not identically zero, finite moments of all orders, compact or non compact supports, and at least one of them having an infinite number of points in its support. We show that for any linear operator T on the space of polynomials with complex coefficients and any integer $n \geq 0$, there is a constant $\gamma_n(T) \geq 0$, such that

$$\|Tp\|_S \leq \gamma_n(T)\|p\|_S,$$

for any polynomial p of degree $\leq n$, where $\gamma_n(T)$ is independent of p, and

$$\|p\|_S = \left\{ \int |p(x)|^2 d\mu_0(x) + \int |p'(x)|^2 d\mu_1(x) \right\}^{\frac{1}{2}}.$$

We find a formula for the best possible value ${}_n(T)$ of $\gamma_n(T)$ and inequalities for ${}_n(T)$. Also, we give some examples when T is a differentiation operator and (μ_0, μ_1) is a vector of orthogonalizing measures for classical orthogonal polynomials.

Schrödinger propagators; Strichartz estimate via Regularisation technique

P. K. Ratnakumar

Abstract

With any self adjoint differential operator $\mathcal{L}$ having the spectral decomposition $\mathcal{L} = \int_{\lambda \in E} \lambda \, dP_\lambda$ we can associate a one parameter oscillatory group $\{e^{-it\mathcal{L}} : t \in \mathbb{R}\}$, called the Schrödinger propagator for $\mathcal{L}$, given by

$$e^{-it\mathcal{L}} = \int_E e^{-it\lambda} dP_\lambda.$$

Here dP_λ denotes the spectral projection for $\mathcal{L}$; i.e., a projection valued measure on the spectrum E of $\mathcal{L}$. Here we consider the case of operators $\mathcal{L}$ with discrete spectrum, in which case, the above integral reduces to an infinite series of the form

$$e^{-it\mathcal{L}} = \sum_k e^{-it\lambda_k} P_k.$$

Oscillatory group of the above form arises in the study of Schrödinger equation for $\mathcal{L}$:

$$i\partial_t u(x,t) - \mathcal{L}u(x,t) = 0, \qquad u(x,0) = f(z), \quad x \in \Omega, \ t > 0,$$

where Ω is $\mathbb{R}^n$ or a submanifold of $\mathbb{R}^m$ for some $m \geq 1$. In fact for $f \in L^2(\Omega)$, the solution to this initial value problem is given by $u(x,t) = e^{-it\mathcal{L}} f(x)$.

Here we discuss a regularisation technique to prove Strichartz estimate for the Schrödinger propagator for a class of differential operators $\mathcal{L}$ with discrete spectrum, which includes the Hermite and special Hermite operators as special cases.

L^p Wiener-Tauberian theorems for $M(2)$

Rama Rawat

Abstract

We prove two sided and one sided analogues of the Wiener-Tauberian theorem for the Euclidean motion group, $M(2)$.

This is a joint work with E. K. Narayanan.

End point estimates for d-plane transform of radial functions

Swagato Ray

Abstract

We will discuss about the End point estimates of the totally geodesic d-dimensional Radon transform of radial functions on real hyperbolic spaces and the sphere. These results can be thought of as analogues of
a) End point estimate for the d-plane transform of radial functions on Euclidean spaces.
b) End point estimate for the Abel transform on noncompact rank one symmetric spaces.

This is a joint work with Ashisha Kumar.

Analysis on nilpotent Gelfand pairs and Hadamard's lemma

Fulvio Ricci

Abstract

One version of Hadamard's lemma states that if a C^∞ function on $\mathbb{R}^n$ has moments vanishing moments up to degree k, then it is a sum of $(k+1)$-th order derivatives of C^∞ functions. The same holds with "C^∞" replaced by "Schwartz". We discuss variants of this statement for K-invariant Schwartz functions on a nilpotent group N, when $(N \rtimes K, K)$ is a Gelfand pair. This is part of a joint work with V. Fischer and O. Yakimova.

Funk, Cosine, and Radon Transforms on Matrix Spaces and Grassmannians

Boris Rubin

Abstract

The Funk transform takes a function f on the unit sphere in the 3-dimensional Euclidean space to a function on the set of great circles as an integral of f over the corresponding great circle. This transform can be regarded as a member of the analytic family of the cosine transforms on the sphere. We extend basic facts about the Funk transform and related analytic families of cosine and sine transforms to the more general context for Stiefel or Grassmann manifolds. Recent results on Radon transforms of L^p functions on matrix spaces and affine Grassmannians are also presented.

The Fourier algebra in the cb-multiplier norm

Volker Runde

Abstract

As the predual of the group von Neumann algebra, the Fourier algebra is a completely contractive Banach algebra. Every function in the Fourier algebra can be viewed as a completely bounded (cb) multiplier. This induces another norm on the Fourier algebra, the cb-multiplier norm, which is not equivalent to the given norm on the Fourier algebra unless the group is amenable. The completion of the Fourier algebra in the cb-multiplier norm is again a completely contractive Banach algebra, whose behaviour can be quite different from the Fourier algebra in its original norm. I shall give a survey with focus on recent results obtained by B. E. Forrest, N. Spronk, and myself.

Pseudo-differential operators on compact Lie groups

Michael Ruzhansky

Abstract

The theory of (pseudo-differential) operators on manifolds usually relies on local representations of operators in local coordinates, thus often ignoring global geometric and other information that is often available. We will present a new approach to pseudo-differential operators exploring global symmetries of the underlying space and yielding a globally defined full symbol. We will describe some applications to the global hypoellipticity and to partial differential equations. The talk will be based on the joint work with Ville Turunen (Helsinki).

An alternative well-posedness property and static spacetimes with naked singularities

Marcela Sanmartino

Abstract

In this work we show that the Cauchy problem for wave propagation in some static spacetimes presenting a singular time-like boundary is well posed,if we only require the waves to have finite energy, although no boundary condition is required.

This feature does not come from essential self-adjointness, which is false in these cases, but from a different phenomenon that we call the alternative well-posedness property, whose origin is due to the degeneracy of the metric components near the boundary. We also state that, although no boundary condition may be imposed on waves, they do have a non trivial boundary behaviour that we describe; finally we characterize the elliptic operators having such properties, among the class of divergence second-order operators.

Sequence characterizing eigenfunctions of Laplacian for Riemannian symmetric spaces

Rudra P. Sarkar

Abstract

We shall try to characterize some eigenfunctions of the Laplace-Beltrami operator of a rank one Riemannian symmetric space of noncompact, type through a sequence of measurable functions satisfying a uniform norm-estimate.

Spectral theory of Beurling-Fourier algebras on compact groups

Nico Spronk

Abstract

For a compact group G, I will define the Beurling-Fourier algebras $A_\omega(G)$ on G, for weights $\omega : \widehat{G} \to \mathbb{R}^{>0}$. The classical Fourier algebra of G corresponds to the case where ω is the constant weight 1. When G is abelian, $A_\omega(G) = \ell^1(\widehat{G}, \omega)$, the classical Beurling algebra on $\widehat{G}$. To describe the spectrum of $A_\omega(G)$, we require an abstract Lie theory which is built from Krein-Tannaka duality, and was formalized separately by McKennon, and Cartwright and McMullen, in the '70s. This Lie theory allows us to develop the complexification $G_\mathbb{C}$, even for non-Lie G. The Gelfand spectrum G_ω can always be realized as a subset of $G_\mathbb{C}$.

We consider the following questions. When, for a symmetric weight ω, is $A_\omega(G)$ symmetric? When is $A_\omega(G)$ regular on G? Can we gain information on sets of spectral synthesis in G_ω?

This is part of joint work, with Jean Ludwig and Lyudmila Turowska. These algebras have also been developed, along different lines, by H.H. Lee and E. Samei.

Harmonic analysis of Laguerre expansions: recent progress

K. Stempak

Abstract

We survey recent results concerning harmonic analysis of different types of Laguerre function expansions. In particular we discuss the following issues: maximal operators, square functions, Riesz transforms and Hardy spaces.

Spherical expansions of zonal functions on the unit sphere in $\mathbb{C}^n$

Aleksander Strasburger

Abstract

In this talk we address the problem of spherical expansions of functions on odd dimensional spheres, i.e. spheres in $\mathbb{C}^n$. For complex zonal functions, i.e. functions invariant under $\mathbf{SU}(n-1)$ — the complex isotropy group of a point, we obtain explicit formulae for expansion coefficients. They are expressed as a series comprising of Taylor coefficients of the profile corresponding to the zonal function. As an application, we show how to deduce the spherical harmonic expansion of the Poisson - Szegö kernel for the unit ball, obtained originally by Folland. In this case summation of the series rests on the use of Pfaff–Saalschütz formula for summing up the hypergeometric series.

This is based in part on the Ph. D. Thesis of A. Bezubik, written under the supervision of the speaker.

Projections in L^1-Algebras of Locally Compact Groups

Keith F. Taylor

Abstract

If G is a locally compact group, a projection in $L^1(G)$ is a self-adjoint idempotent. After briefly reviewing the descriptions of projections for abelian, compact and unimodular groups, we will discuss the construction of projections in certain classes of non-unimodular groups emphasizing the role of the topology on the dual (the space of equivalence classes of irreducible unitary representations) of the group in understanding projections. We will relate the constructions to multi-dimensional generalizations of the continuous wavelet transform, such as the continuous shearlet transform.

A uniqueness theorem for the Schrödinger equation on H-type groups

S. Thangavelu

Abstract

We prove a uniqueness theorem for solutions of the Schrödinger equations associated to sublaplacians on H-type groups. This is really 'old wine in a new bottle' or 'cold coffee in a new mug'. This is joint work with Salem Ben Said'

On the restriction of the metaplectic representation over a p-adic field to maximal compact subgroups and maximal anisotropic tori

Pierre Torasso

Abstract

On the restriction of the metaplectic representation over a p-adic field to maximal compact subgroups and maximal anisotropic tori Pierre Torasso The metaplectic representation is a basic object in representation theory which plays a crucial role in the construction of representations of general algebraic groups over a local field (real or p-adic). In order to study restriction of discrete series to closed subgroups of solvable groups, it is important to compute the restriction of the metaplectic representation to anisotropic tori of the metaplectic group. Except for SL_2, this was not known in the p-adic case. In this case, we prove that the restriction of the metaplectic representation to the maximal compact subgroups and maximal anisotropic tori is multiplicity free and give explicit description of the irreducible representations and characters occurring. This is joint work with Khemais Maktouf.

Restriction Theorems for Surfaces with Vanishing Curvatures

Ana Vargas

Abstract

We prove some bilinear restriction estimates for conic type surfaces with more than one vanishing curvature. As corollary, we show new restriction estimates for some conic surfaces.