Weighted weak type bound for rough maximal singular integrals near L^1

Ankit Bhojak
(Indian Institute of Technology Kanpur)

Let $\Omega : \mathbb{S}^{d-1} \to \mathbb{R}$ with $\int \Omega(\theta)\, d\theta = 0$. The rough singular intergral T_Ω is defined as

$$T_\Omega f(x) = p.v. \int_{\mathbb{R}^d} \frac{\Omega(y')}{|y|^d} f(x-y)dy.$$

It is well known that T_Ω is bounded from $L^p(w)$ to $L^p(w)$, $1 < p < \infty$ for $\Omega \in L^\infty(\mathbb{S}^{d-1})$ and $w \in A_p$. It was shown that the dependence of the $L^2(w)$ boundedness of T_Ω on the A_2 characteristic is quadratic, although it is not known whether the dependence is sharp. For the endpoint $p = 1$, Seeger (1996) showed that $T_\Omega : L^1 \to L^{1,\infty}$. Li et al (2019) showed the corresponding quantitative bound:

$$\|T_\Omega\|_{L^1(w)\to L^{1,\infty}(w)} \lesssim [w]_{A_1}[w]_{A_\infty} \log([w]_{A_\infty} + 1), \text{ for } w \in A_1.$$

It is well known that the corresponding maximal singular integral T^*_Ω given by

$$T^*_\Omega f(x) = \sup_{\epsilon>0} \left| \int_{|y|>\epsilon} \frac{\Omega(y')}{|y|^d} f(x-y)dy \right|$$

is bounded from $L^p(w)$ to $L^p(w)$, $1 < p < \infty$ for $\Omega \in L^\infty$ and $w \in A_p$. Recently, the dependence of L^2 boundedness of T^*_Ω on the A_2 characteristic was also shown to be quadratic. However, it is an longstanding open problem to show the boundedness of T^*_Ω at the endpoint $p = 1$. We have shown that for $w \in A_1$ and $\Omega \in L^\infty(\mathbb{S}^{d-1})$, T^*_Ω is of weak type $L \log\log L(w)$ with weight dependence $[w]_{A_1}[w]_{A_\infty} \log([w]_{A_\infty} + 1)$, which is same as the best known constant for T_Ω. This result is new even in the unweighted case. (This work is a collaboration with Parasar Mohanty.)

Wavelet system and Muckenhoupt $\mathcal{A}_2$ condition on the Heisenberg group

Arati Shashi
(NBHM Post-Doctoral Fellow, IISER Thiruvananthapuram)

Let $\mathbb{H}^n$ denote the Heisenberg group. It is shown that under certain conditions the wavelet system $\{\psi_{j,k,l,m} : k, l \in \mathbb{Z}^n,\, j, m \in \mathbb{Z}\}$ on $\mathbb{H}^n$ arising from integer translations and nonisotropic dilations forms a Schauder basis for its closed linear span in $L^2(\mathbb{H}^n)$ if and only if the function $\sum_{r\in\mathbb{Z}} \|\widehat{\psi}(\cdot + r)\|^2_{\mathcal{B}_2} |\cdot + r|^n$ satisfies the Muckenhoupt $\mathcal{A}_2$ condition, where $\mathcal{B}_2$ denotes the class of Hilbert-Schmidt operators on $L^2(\mathbb{R}^n)$. This is a joint work with R. Radha.

On L^p-L^q multiplier

Arup Kumar Maity
(Harish-Chandra Research Institute)

In this talk I will discuss the L^p-L^q Fourier multipliers on $\mathbb{C}^n$ which arise as twisted convolution of two functions.

On some generalized inequalities for the integral operators of Hardy-Steklov type

Duranta Chutia
(Tezpur University)

Let us consider the integral operators of Hardy-Steklov type $\mathcal{I}$, for a non-negative measurable function f on $-\infty \le a < b \le \infty$, defined by

$$\mathcal{I}f(t) = h(t)\int_{\alpha(t)}^{\beta(t)} K(t,z)f(z)w(z)dz, \tag{1}$$

where $\alpha, \beta : (a,b) \to \mathbb{R}$ are continuous and increasing functions satisfying $\alpha(z) \le \beta(z)$ for each $z \in (a,b)$, h and w are positive measurable functions, and let the kernel $K(t,z)$ defined on $\{(t,z); \alpha(t) \le z \le \beta(t)\}$ satisfies the following conditions:

(a) $K(t,z) \ge 0$.
(b) $K(t,z)$ is non-decreasing in t and non-increasing in z.
(c) Suppose that $M \ge 1$ be a constant independent of t, z and τ such that

$$K(t,z) \le M\Big[K(t,\beta(\tau)) + K(\tau,z)\Big],$$

where $\tau \le t$ and $\alpha(t) \le z \le \beta(\tau)$.

In this short talk, we study the boundedness of the operator (1) in Orlicz space setting using the idea of N- functions. We provide suitable conditions on the weights ω, ρ, ϕ and ψ for which the integral operators of Hardy-Steklov type, $\mathcal{I}$ satisfies weak type mixed modular inequalities of the form

$$\mathcal{U}^{-1}\Big(\int\limits_{\{\mathcal{I}f>\gamma\}} \mathcal{U}(\gamma\omega)\rho\Big) \le \mathcal{V}^{-1}\Big(\int \mathcal{V}(Cf\phi)\psi\Big),$$

where $\mathcal{U}$ is strictly increasing and positive, and $\mathcal{V}$ is an N-function. We also establish the following mixed integral inequalities of the extra-weak type under appropriate conditions on the weights ω, ϕ and ψ.

$$\omega\Big(\{\mathcal{I}f > \gamma\}\Big) \le \mathcal{U}\circ\mathcal{V}^{-1}\Big(\int \mathcal{V}\Big(\frac{Cf\phi}{\gamma}\Big)\psi\Big).$$

Further, we discuss the above two integral inequalities for the adjoint of the integral operators of Hardy-Steklov type defined by

$$\tilde{\mathcal{I}}f(t) = w(t)\int_{\beta^{-1}(t)}^{\alpha^{-1}(t)} K(z,t)f(z)h(z)dz.$$

Pointwise Fatou theorem and its converse for solutions of the heat equation on a stratified Lie group

Jayanta Sarkar
(ISI Kolkata)

A classical result due to Fatou (1906) says that Poisson integral $P[\mu]$ of a measure μ on $\mathbb{R}$ has nontangential limit L at each points x_0 of differentiability of the distribution function F of μ with $L = F'(x_0)$. In 1943, Loomis proved that the converse is false in general but holds true for positive measures. Later, Ramey and Ullrich (1988) extended Fatou-Loomis theorem in higher dimensions for positive measures. In 1990, Brossard and Chevalier generalized this result for more general class of measures. In this talk, we plan to discuss analogues of these results for solutions of heat equations on stratified Lie groups.

Weighted estimates for maximal product of spherical averages.

Kalachand Shuin
(Iiser Mohali)

The spherical averages of a continuous function f is defined by

$$A_t f(x) := \int_{\mathbb{S}^{n-1}} f(x-ty)d\sigma(y), \text{ for } t>0.$$

Spherical averages often make their appearance as a solution of partial differential equations. For instance the average $u(x,t) = \frac{1}{4\pi}\int_{\mathbb{S}^2} tf(x-ty)\, d\sigma(y)$ is a solution of the wave equation

$$\triangle_x u(x,t) = \frac{\partial^2 u}{\partial t^2}(x,t),$$
$$u(x,0) = 0,\ \frac{\partial u}{\partial t}(x,0) = f(x),$$

in $\mathbb{R}^3$. In 1976, E.M. Stein proved that the spherical maximal operator $M_{\text{full}}f = \sup_{t>0}|A_t f|$ is bounded on $L^p(\mathbb{R}^n)$ if and only if $p > \frac{n}{n-1}$ for $n \geq 3$. Later, Bourgain extended the above boundedness result to dimension $n = 2$. In the spirit of bilinear Hardy-Littlewood maximal function (considered by A. Lerner et al. 2009), we have considered the following operator

$$\mathcal{M}_{\text{full}}(f_1, f_2)(x) := \sup_{t>0}|A_t f_1(x) A_t f_2(x)|.$$

We have also considered the lacunary maximal function as follows

$$\mathcal{M}_{\text{lac}}(f_1, f_2)(x) := \sup_{j\in\mathbb{Z}}|A_{2^j} f_1(x) A_{2^j} f_2(x)|.$$

In this talk, I shall discuss sparse domination and weighted estimates of the above mentioned maximal product of spherical averages with respect to genuine bilinear weights. The following theorem is one of our main result.

Theorem 1. *Let $n \geq 2$. For $i = 1,2$, let $(\frac{1}{r_i}, \frac{1}{s_i})$ be in the interior of L_n (respectively F_n). Assume that $t := \frac{s_1 s_2}{s_1+s_2-s_1 s_2} > 1$. Then for all $\vec{q} = (q_1, q_2)$, $\frac{1}{q} = \frac{1}{q_1} + \frac{1}{q_2}$ with $r_i \leq q_i$, $i = 1,2$, and $t' > q$, the operator $\mathcal{M}_{\text{lac}}$ (respectively $\mathcal{M}_{\text{full}}$) extends to a bounded operator from $L^{q_1}(w_1) \times L^{q_2}(w_2) \to L^q(w)$, i.e.,*

$$\|\mathcal{M}(f_1, f_2)\|_{L^q(w)} \leq C([\vec{w}]_{A_{\vec{q},\vec{r}}}) \prod_{i=1}^{2} \|f_i\|_{L^{q_i}(w_i)},$$

where $\mathcal{M} := \mathcal{M}_{\text{lac}}$ (respectively $\mathcal{M}_{\text{full}}$), $\vec{w} = (w_1, w_2) \in A_{\vec{q},\vec{r}}$ with $\vec{r} = (r_1, r_2, t)$ and $C([\vec{w}]_{A_{\vec{q},\vec{r}}})$ is a constant depending on the characteristic of the weight.

On the boundedness of continuous wavelet transform involving fractional Hankel transform on Gelfand-Shilov spaces

Kanailal Mahato
(Department of Mathematics, Banaras Hindu University)

We give characterization results of the fractional Hankel transform as well its inverse on some Gelfand–Shilov spaces of type W and S. Furthermore, we derive the boundedness properties of wavelet transforms involving the fractional Hankel transform on certain suitably constructed spaces of type W as well as S.

Kato-Ponce estimates for fractional sublaplacians

Luz Roncal
(Basque Center for Applied Mathematics - BCAM)

We give a proof of commutator estimates for fractional powers of the sublaplacian on the Heisenberg group. Our approach is based on pointwise and L^p estimates involving square fractional integrals and Littlewood-Paley square functions. Joint work with Luca Fanelli.

Characterization of Fourier transform of H-valued functions on the Real line

Md Hasan Ali Biswas
(IIT Madras)

We characterize the Fourier transform for functions belonging to $\mathcal{L}^2(\mathbb{R}, H)$, where H denotes a Hilbert C^*-module. But in the case of functions belonging to $L^1(\mathbb{R}, H)$ we obtain a similar result when H is a separable Hilbert space.

Theorem 2. *Let $(H, \langle \cdot, \cdot \rangle)$ be a Hilbert A-module with countable orthonormal basis $\{e_1, e_2, ...\}$ and $T : \mathcal{L}^2(\mathbb{R}, H) \to \mathcal{L}^2(\mathbb{R}, H)$ be a bounded, linear and surjective map satisfying*
(i) *$T((f \star g) \odot h)(\xi) = \langle Tf(\xi), Tg(-\xi) \rangle Th(\xi)$, for $f, g, h \in \mathcal{L}^2(\mathbb{R}, H),\ (f \star g) \odot h \in \mathcal{L}^2(\mathbb{R}, H),\ \xi \in \mathbb{R}$,*
(ii) *$T(\tau_s f)(\xi) = e^{-2\pi i s \xi} Tf(\xi),\ f \in \mathcal{L}^2(\mathbb{R}, H),\ s, \xi \in \mathbb{R}$,*
(iii) *T is of the form $S \otimes I$ for some $S \in B(L^2(\mathbb{R}))$ and I is the identity operator on H,*
(iv) *$T(\tilde{f})(-\xi) = Tf(\xi),\ f \in L^2(\mathbb{R}, H),\ \xi \in \mathbb{R}$, where $\tilde{f}(x) = f(-x)$.*
Then $\langle Tf(\xi), Tg(\xi) \rangle = \langle \hat{f}(\xi), \hat{g}(\xi) \rangle, \quad f, g \in \mathcal{L}^2(\mathbb{R}, H),\ \xi \in \mathbb{R}$.

Sharp Adams type inequalities for the fractional Laplace-Beltrami operator on noncompact symmetric spaces

Mithun Bhowmik
(Indian Institute of Science, Bangalore)

We establish sharp Adams type inequalities on Sobolev spaces $W^{\alpha, n/\alpha}(X)$ of any fractional order $\alpha < n$ on Riemannian symmetric space X of noncompact type with dimension n and of arbitrary rank. We also establish sharp Hardy-Adams inequalities on the Sobolev spaces $W^{n/2,2}(X)$. For the real hyperbolic spaces, such results were recently obtained by J. Li et al. (Trans. AMS, 2020). We use Fourier analysis on the symmetric spaces to obtain these results.

Large time behaviour of heat propagator

Muna Naik
(Harish-Chandra Research Institute, Prayagraj)

Let S be a Damek–Ricci space and Δ be the Laplace–Beltrami operator of S. In this talk we will try to explore the behaviour of heat propagator in S in large time to illustrate the differences with the corresponding results in $\mathbb{R}^n$. In particular we shall talk about the relation between the limiting behaviour of the ball-averages as radius tends to ∞ and that of the the heat propagator as time goes to ∞ and use this relation for characterization of eigenfunctions of Δ.

OSCILLATORY INTEGRALS AND FINE-SCALE STATISTICS

Niclas Technau
(California Institute of Technology)

In the late 1990s, Rudnick and Sarnak popularised the study of m-point correlation functions. These functions measure randomness of sequences, and are partially motivated by the BerryTabor conjecture in quantum chaos. The present talk explains how m-point correlation functions can be analysed in terms of oscillatory integrals, and with techniques from discrete harmonic analysis. In particular, I will report on recent joint work with Athanasios Sourmelidis, and Christopher Lutsko as well as a follow up paper with Christopher Lutsko on the m-point correlations of monomial sequences of the shape $(\alpha n^{\theta})n$ where $\alpha, \theta > 0$ are fixed.

An uncertainty principle on the Heisenberg group

Pritam Ganguly
(Indian Institute of Science)

An uncertainty principle due to Ingham investigates the best possible decay admissible for the Fourier transform of a function on $\mathbb{R}$ which vanishes on a nonempty open set. One way to prove Ingham type result on $\mathbb{R}^n$ is to use a theorem of Chernoff, which provides a sufficient condition for a smooth function on $\mathbb{R}^n$ to be quasi-analytic in terms of a Carleman condition involving powers of the Laplacian. In this talk, we plan to discuss a Chernoff type theorem for the full Laplacian on the Heisenberg group and use it to show analogues of Ingham's theorem for the operator-valued group Fourier transform on the same group. This is a joint work with S.Thangavelu.

Group of isometries of an infinite dimensional hyperbolic space

Rachna Aggarwal
(University of Delhi)

In the present article, geometry of the infinite dimensional hyperbolic space has been studied. We will consider the group of isometries of the Hilbert ball equipped with the Carathéodory metric and intend to study some dynamical aspects of this group. Properties of some special subclasses of this group are explored along with the discussion on conjugacy classes within the group.

Group of isometries of an infinite dimensional hyperbolic space

Rajula Srivastava
(University of Wisconsin Madison)

We discuss $L^p \to L^q$ estimates for local maximal operators associated with dilates of codimension two spheres in Heisenberg groups, sharp up to two endpoints. The proof shall be reduced to estimates for standard oscillatory integrals of Carleson-Sjölin-Hörmander type, relying on the maximal possible number of nonvanishing curvatures for a cone in the fibers of the associated canonical relation. The results can be applied to improve currently known bounds on sparse domination for global maximal operators.

On sharp weighted L_p-estimates for pseudo-multipliers associated to Grushin operators

Riju Basak
(IISER Bhopal)

In this talk, I shall discuss weighted L_p -boundedness results for a class of pseudo-multipliers associated to Grushin operators. Under some H örmander-type con-ditions on symbol functions, we establish pointwise domination of pseudo-multipliers by sparse operators. This is a joint work with Sayan Bagchi, Rahul Garg, and Abhishek Ghosh.

On some analogues of Beurling's theorem

Santanu Debnath
(University of Calcutta)

A classical result of Arne Beurling says that if the Fourier transform of a non-zero integrable function on the real line has certain exponential decay then the function can not vanish on a set of positive Lebesgue measure. In this talk we present some several variable analogues of this theorem.

Twisted B-splines in the complex plane

Santi Ranjan Das
(IIT Madras)

We introduce a new class of twisted B-splines and study some properties of these B-splines. We also investigate the system of twisted translates and the wavelets corresponding to these twisted B-splines.

Theorem 3. *The twisted B-splines ϕ_1 and ϕ_2 satisfy the following partition-of-unity-line property:*

$$\iint_{\mathbb{R}^2} \sum_{k,l\in\mathbb{Z}} T^t_{(k,l)}\phi_1(x,y)\; dx\,dy = 1$$

and

$$\iint_{\mathbb{R}^2} \sum_{k,l\in\mathbb{Z}} T^t_{(k,l)}\phi_2(x,y)\; dx\,dy = C_{\phi_2},$$

where the constant $C_{\phi_2} \approx 0.000160507/\pi^2$. Moreover, the functions L_n defined by

$$\iint_Q L_n(u,v)du\,dv := \iint_{\mathbb{R}^2} \sum_{k,l\in\mathbb{Z}} T^t_{(k,l)}\phi_{n+1}(x,y)\; dx\,dy,$$

can be recursively computed in terms of their lower orders via

$$L_{n+1}(u,v) = \iint_Q e^{\pi i(ut-vs)} L_n(u+s, v+t)\; ds\,dt,$$

where $Q := [0,1]\times[0,1]$.

Pseudo-differential operators, Wigner transform and Weyl transform on the Similitude group, $\mathbb{SIM}(2)$

Santosh Kumar Nayak
(IIT Delhi)

In this paper we study pseudo-differential operators with operator valued symbols on the Similitude group, $\mathbb{SIM}(2)$. We characterize the unique infinite-dimensional, unitary, irreducible representations of $\mathbb{SIM}(2)$ and using that the Fourier inversion formula is given. The operator valued Wigner transform and the Weyl transform are also studied here. Furthermore we also obtain a necessary and sufficient condition on the symbols for which the pseudo-differential operators on $\mathbb{SIM}(2)$ are Hilbert-Schmidt and trace class operators. Finally, we also give the trace formula for these operators.

1. Selected Results

Theorem 4. *Let $\sigma : \mathbb{SIM}(2)\times\widehat{\mathbb{SIM}(2)} \to B(L^2(\mathbb{R}^2))$ be an operator valued symbol such that for all $(b,a,\theta) \in \mathbb{SIM}(2), D\sigma(b,a,\theta)$ is an integral operator from $L^2(\mathbb{R}^2)$ to $L^2(\mathbb{R}^2)$ in which the kernel belonging to $L^2(\mathbb{R}^2 \times \mathbb{R}^2)$. Then the pseudo diferential operator $T_\sigma : L^2(\mathbb{SIM}(2)) \to L^2(\mathbb{SIM}(2))$ is a bounded operator.*

Theorem 5. *Let $\sigma : \mathbb{SIM}(2) \times \widehat{\mathbb{SIM}(2)} \to S_2$ be an operator valued symbol such that the hypothesis of Theorem 4 is satisfied. Then the corresponding pseudo-differential operator $T_\sigma : L^2(\mathbb{SIM}(2)) \to L^2(\mathbb{SIM}(2))$ is a Hilbert-Schmidt operator if and only if*

$$D\sigma(b,a,\theta,\pi) = \pi(b,a,\theta)W_{\tau_{\alpha(b,a,\theta)}}, (b,a,\theta) \in \mathbb{SIM}(2),$$

where $\tau_{\alpha(b,a,\theta)}(x,y) = \mathcal{F}_2^{-1}TK_{\alpha(b,a,\theta)}(x,y)$, T is the Twisting operator, $W_{\tau_{\alpha(b,a,\theta)}}$ is a Weyl transform of $\tau_{\alpha(b,a,\theta)}$,

$$K_{\alpha(b,a,\theta)}(x,y) = \begin{cases} \mathcal{F}_1^{-1}\frac{\alpha(b,a,\theta)}{a}\Big(x, R_\theta y.\overline{x}, \cos^{-1}\Big(\frac{x.y}{\|x\|\|y\|}\Big)\Big), & \text{if } x\neq 0, y\neq 0,\\ 0, & \text{otherwise},\end{cases}$$

and $\alpha : \mathbb{SIM}(2) \to L^2(\mathbb{SIM}(2))$ is a weakly continuous mapping for which

$$\int\limits_{\mathbb{SIM}(2)} \|\alpha(b,a,\theta)\|^2_{L^2(\mathbb{SIM}(2))}\frac{dbdad\theta}{a^3} < \infty.$$

Theorem 6. *Let $\sigma : \mathbb{SIM}(2) \times \widehat{\mathbb{SIM}(2)} \to S_2$ be a symbol satisfying the hypothesis of uniqueness theorem. Then the pseudo-differential operator $T_\sigma : L^2(\mathbb{SIM}(2)) \to L^2(\mathbb{SIM}(2))$ is a trace class operator if and only if*

$$D\sigma(b,a,\theta,\pi) = \pi(b,a,\theta)W_{\tau_{\alpha(b,a,\theta)}},$$

where $\alpha : \mathbb{SIM}(2) \to L^2(\mathbb{SIM}(2))$ is a mapping such that the conditions of Theorem 5 are satisfied,

$$\alpha(b,a,\theta)(b',a',\theta') = \int_{\mathbb{SIM}(2)} \alpha_1(b,a,\theta)(b'',a'',\theta'')\alpha_2(b'',a'',\theta'')(b',a',\theta')\frac{db''da''d\theta''}{a''^3}$$

for all $(b,a,\theta),(b',a',\theta') \in \mathbb{SIM}(2)$, and $\alpha_i : \mathbb{SIM}(2) \to L^2(\mathbb{SIM}(2))$, $i = 1,2$, are such that

$$\int_{\mathbb{SIM}(2)} \|\alpha_i(b,a,\theta)\|^2_{L^2(\mathbb{SIM}(2))}\frac{dbdad\theta}{a^3} < \infty,$$

for $i = 1,2$. Moreover, if $T_\sigma : L^2(\mathbb{SIM}(2)) \to L^2(\mathbb{SIM}(2))$ is a trace class operator, then its trace

$$\begin{aligned} Tr(T_\sigma) &= \int\limits_{\mathbb{SIM}(2)} \alpha(b,a,\theta)(b,a,\theta)\frac{dbdad\theta}{a^3} \\ &= \int\limits_{\mathbb{SIM}(2)}\int\limits_{\mathbb{SIM}(2)} \alpha_1(b,a,\theta)(b',a',\theta')\alpha_2(b',a',\theta')(b,a,\theta)\frac{db'da'd\theta'}{a'^3}\frac{dbdad\theta}{a^3}. \end{aligned}$$

Benedicks-Amrein-Berthier theorem for the Heisenberg motion group

Somnath Ghosh
(TIFR CAM)

A fundamental result by Benedicks states that if $f \in L^1(\mathbb{R}^n)$, then both the sets $\{x \in \mathbb{R}^n : f(x) \neq 0\}$ and $\{\xi \in \mathbb{R}^n : \hat{f}(\xi) \neq 0\}$ cannot have finite Lebesgue measure, unless $f = 0$. For the Heisenberg group $\mathbb{H}^n$, novelty is obtained by the following result investigated by Narayanan and Ratnakumar. If $f \in L^1(\mathbb{H}^n)$ is supported on $B \times \mathbb{R}$, where $B \subset \mathbb{C}^n$ is compact, and $\hat{f}(\lambda)$ has finite rank for each λ, then $f = 0$.

Since $(\mathbb{H}^n, U(n))$ is a Gelfand pair, an exact analogue of the Heisenberg group result is not true for the Heisenberg motion group. In this talk, we shall reach out to a variant of such result by considering finite rank condition on the Weyl transform, which is non-zero for finitely many Fourier-Wigner pieces, together with the finite support condition. (Joint work with Dr. Rajesh Kumar Srivastava).

REPRODUCING FORMULA ON NILPOTENT LIE GROUPS

Sudipta Sarkar
(Department of Mathematics, IIT Indore)

Let G be a connected, simply connected, nilpotent Lie group whose irreducible unitary representations are square-integrable modulo the center. For a compatible subgroup Γ of G, we obtain characterizations for Γ-translation generated systems satisfying reproducing formulas by the action of a countable collection of functions $\mathcal{A}$ in $L^2(G)$. The current study extends the Euclidean case, and it occurs within the set up of continuous frames, which means the resulting reproducing formulas are given in terms of integral representations instead of discrete sums. Using the Plancherel transform followed by periodization, we obtain results based on the bracket analysis for single generated systems and the range function techniques for associated countable generators. As an illustration of our result, we discuss the reproducing formulas for the d-dimensional Heisenberg group $\mathbb{H}^d$ in connection with the orthonormal Gabor systems of $L^2(\mathbb{R}^d)$.

Result: Let us assume that G be an SI/Z nilpotent Lie group with center Z. For a chosen basis $\{X_1, X_2, \ldots, X_n\}$ of the corresponding Lie algebra $\mathfrak{g}$, we consider the center Z identified with $\mathbb{R}^r$ $(r < n)$ as follows: $Z = \exp \mathbb{R}X_1 \exp \mathbb{R}X_2 \ldots \exp \mathbb{R}X_r$.
• $\Gamma = \Gamma_1\Gamma_0 = \{\gamma_1\gamma_0 : \gamma_1 \in \Gamma_1, \gamma_0 \in \Gamma_0\}$, where $\Gamma_1 \subseteq \exp \mathbb{R}X_{r+1} \ldots \exp \mathbb{R}X_n$ (not necessarily discrete) and the integer lattice $\Gamma_0 = \exp \mathbb{Z}X_1 \ldots \exp \mathbb{Z}X_r$.
• For a countable family $\mathcal{A} = \{\varphi_t : t \in \mathcal{N}\} \subset L^2(G)$, the left translation generated system $\mathcal{E}^\Gamma(\mathcal{A}) = \{L_\gamma\varphi : \gamma \in \Gamma, \varphi \in \mathcal{A}\}$ and its span closure $\mathcal{S}^\Gamma(\mathcal{A}) = \overline{\text{span}}\, \mathcal{E}^\Gamma(\mathcal{A})$.
• The *fiberization map* is an isometric isomorphism defined by:

$$\mathcal{F} : L^2(G) \to L^2(\mathbb{T}^r; \ell^2(\mathbb{Z}^r, \mathcal{HS}(L^2(\mathbb{R}^d))))$$
$$\mathcal{F}f(\sigma)(j) = \widehat{f}(\sigma + j)|\boldsymbol{Pf}(\sigma + j)|^{1/2},$$

for all $f \in L^2(G)$, a.e., $\sigma \in \mathbb{T}^r$.
• An ℓ^2-valued *range function* on $\mathbb{T}^r$ is a mapping

$$J : \mathbb{T}^r \to \{\text{closed subspaces of } \mathbb{E}ll\}.$$

With the help of above notion we can discuss global reproducing formula locally for a.e. $\sigma \in \mathbb{T}^r$.

Theorem 7. *Let the system $\mathcal{E}^\Gamma(\mathcal{A})$ be a frame for the translation invariant space $\mathcal{S}^\Gamma(\mathcal{A})$ in $L^2(G)$, and also let the another system $\mathcal{E}^\Gamma(\mathcal{A}')$ be a Bessel system, where $\mathcal{A}' = \{\psi_t : t \in \mathcal{N}\} \in L^2(G)$. Then, the following are equivalent :*

(i) *The system $\mathcal{E}^\Gamma(\mathcal{A}')$ satisfying the reproducing formula for $\mathcal{E}^\Gamma(\mathcal{A})$ in $L^2(G)$, i.e.,*

$$f = \sum_{t \in \mathcal{N}} \int_\Gamma \langle f, L_\gamma\psi_t \rangle L_\gamma\varphi_t d\gamma, \ \forall f \in \mathcal{S}^\Gamma(\mathcal{A}).$$

(ii) *For a.e.* $\sigma \in \mathbb{T}^r$, *the system* $\mathcal{A}'(\sigma) = \{\mathcal{F}L_{\gamma_1}\psi_t(\sigma) : \gamma_1 \in \Gamma_1, t \in \mathcal{N}\}$ *satisfying the reproducing formula for the frame* $\mathcal{A}(\sigma) = \{\mathcal{F}L_{\gamma_1}\varphi_t(\sigma) : \gamma_1 \in \Gamma_1, t \in \mathcal{N}\}$ *of* $J(\sigma)$ *in* $\ell^2(\mathbb{Z}^r, \mathcal{HS}(L^2(\mathbb{R}^d)))$, *i.e.,*

$$h = \sum_{t\in\mathcal{N}} \int_{\Gamma_1} \langle h, \mathcal{F}L_{\gamma_1}\psi_t(\sigma)\rangle \mathcal{F}L_{\gamma_1}\phi_t(\sigma) d\gamma_1, \forall h \in J_{\mathcal{A}}(\sigma),$$

where $\mathcal{F}$ *is the fiberization map and* $J(\sigma)$ *is the range function.*

Tensor products of local operator systems

Surbhi Beniwal
(University of Delhi, New Delhi, India)

Joint work with Ajay Kumar and Preeti Luthra

Local operator systems are projective limits of operator systems. We explore several tensor products including local minimal ($lmin$), local maximal ($lmax$), local commuting, and λ-tensor product of local operator systems. A characterization of ($lmin$,$lmax$)-nuclearity in terms of local completely positive factorization property will also be discussed. We also prove that projective limit of operator systems having *completely positive factorization property* (CPFP) is a local operator system having *local completely positive factorization property*

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On bilinear Bochner-Riesz Square function

Surjeet Singh Choudhary
(IISER Bhopal)

For $\alpha > 0$, the bilinear Bochner-Riesz operator of order α in $\mathbb{R}^n$ is defined as

$$\mathcal{B}^\alpha(f,g)(x) = \int_{\mathbb{R}^n} (1 - |\xi|^2 - |\eta|^2)_+^\alpha \hat{f}(\xi)\hat{g}(\eta) e^{2\pi i x\cdot(\xi+\eta)} d\xi d\eta.$$

The Stein's square function associated with bilinear Bochner-Riesz operator is defined as

$$\mathcal{G}^\alpha(f,g)(x) = \left(\int_0^\infty \left| \frac{\partial}{\partial R} \mathcal{B}_R^{\alpha+1}(f,g)(x) \right|^2 R dR \right)^{\frac{1}{2}}.$$

In this talk we will discuss its L^p boundedness properties. This is based on a joint work with Jotsaroop Kaur, Saurabh Shrivastava and Kalachand Shuin.

L^p-BOUNDEDNESS OF PSEUDO DIFFERENTIAL OPERATORS ON RANK ONE RIEMANNIAN SYMMETRIC SPACES OF NONCOMPACT TYPE

Tapendu Rana
(Indian Institute of Technology, Bombay)

For a given function $a(x,\xi)$ on $\mathbb{R}^n \times \mathbb{R}^n$, consider the pseudo differential operator $a(x,D)$ defined by

$$a(x,D)(f) = \int_{\mathbb{R}^n} a(x,\xi)\widehat{f}(\xi)e^{2\pi i x\cdot\xi}d\xi,$$

where $\widehat{f}$ is the Fourier transform of a function f. Let S^0 be the set of all smooth functions $a : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{C}$ satisfies,

$$\left|\partial_x^\beta \partial_\xi^\alpha a(x,\xi)\right| \leq C_{\alpha,\beta}\left(1+|\xi|\right)^{-|\alpha|}$$

for all $x, \xi \in \mathbb{R}^n$ and for all multi indices α and β. Then the following result is well known:

Theorem 8. *For $a \in S^0$, $a(x,D)$ extends to a bounded operator on $L^p(\mathbb{R}^n)$ to itself, for $1 < p < \infty$.*

In this talk, we will discuss an analogue of this result on rank one Riemannian symmetric spaces of noncompact type. This is a joint work with Prof. Sanjoy Pusti.