WORKSHOP ON NUMBER THEORY, NISER

Speaker: Prof. M. Waldschmidt
Title: A survey on Diophantine properties of the Riemann zeta function.
Abstract: Few results are known and many problems are open dealing with the arithmetic properties of the Riemann zeta function at rational integers.

Speaker: Dr. Kaneenika Sinha
Title: Central limit theorems in number theory.
Abstract: The goal of these lectures is to review an important theme at the intersection of probability and number theory, namely central limit theorems in number theory. After reviewing the "prototype" theorem in this theme, namely the classical Erdos-Kac theorem about the prime-omega function, we will survey different types of central limit theorems in the context of a) zeta zeroes for families of curves over finite fields, b) eigenvalues of Hecke operators acting on spaces of cusp forms, and c) eigenvalues of random regular graphs.
For the proof of each of these CLTs, one combines the method of moments with the estimation of specific exponential sums for each of these families.

Speaker: D Surya Ramana
Title: Large sieve

Speaker: Shivansh Pandey
Title: Nonvanishing of kernel functions and Poincare series for Jacobi forms.
Abstract: In this talk we first discuss kernel functions defined by Y. Martin. Then we prove nonvanishing of Fourier coefficients of kernel functions. As a consequence we give results on nonvanishing of L-functions and Poincare series for Jacobi forms.

Speaker: Mrityunjoy Charan
Title: Poincaré series associated to any q series.
Abstract: Recently Brandon Williams studied Poincaré series associated to any q-series whose coefficients grow slowly enough. In this talk,

we define and briefly discuss some properties of this type of Poincaré series. As an application, we discuss the adjoint of higher order Serre derivative maps and some identities involving Fourier coefficients of certain cusp forms.

Speaker: Sandeep E.M
Title: Zero free regions of spectral averages of L-functions.
Abstract: Since the time of Riemann (1859), non-vanishing of L-functions has been an interesting area of research in analytic number theory. Later, once the theory of automorphic forms began developing systematically under the shoulders of Ramanujan, Hecke, Siegel, Selberg (to name a few), number theorists started looking at the L-functions associated to such forms. In this talk, I would describe a recent result on the zero-free region of a weighted average of L-functions of integral weight Hecke cusp forms. If time permits, similar results obtained for those corresponding to (weight zero) Hecke-Maass cusp forms (both of level 1) would also be discussed. The first is a joint work with Dr. Satadal Ganguly.

Speaker: Jagannath Bhanja
Title: Sumsets of dilated integer sets.
Abstract. Let A be a non-empty finite set of integers. For any positive integer k, let $A + k \cdot A = \{a_1 + ka_2 : a_1, a_2 \in A\}$. One of the central problems in additive combinatorics is to estimate the size of the sumset $A + k \cdot A$. It was conjectured by Cilleruelo, Silva, and Vinuesa that for every finite set of integers A with sufficient cardinality, one has $|A + k \cdot A| \geq (k+1)|A| - \lceil k(k+2)/4 \rceil$, and this bound is optimal. For some values of k, this conjecture is already confirmed. In this talk, we shall prove this conjecture under certain conditions on set A and for all positive integers k except for those of the form $2^{m_1}p^{m_2}$ or $2^{m_1}3^{m_2}5^{m_3}$, where p is an odd prime number and $m_i \geq 1$ for $i = 1, 2, 3$.

Speaker: Lalit Vaishya
Title: An upper bound estimate on first ever negative sign of Hecke eigenvalues at integers in sparse set.
Abstract: In this talk, we consider a question concerning the estimation of summatory function of the Fourier coefficients of Hecke eigenforms indexed by a sparse set of integers represented by a primitive integral positive-definite binary quadratic forms Q of fixed discriminant D ¡ 0 with the class number h(D) = 1. As a consequence, we determine the size, in terms of conductor of associated L-function, for

the first sign change of Hecke eigenvalues indexed by the integers which are represented by Q.

Speaker: Nilanjan Bag.
Title: Bounds on bilinear sums of Kloosterman sums.
Abstract: In this talk we shall discuss some elementary arguments to obtain new bounds on bilinear sums with weighted Kloosterman sums which complements those recently obtained by E. Kowalski, P Michel and W. Sawin (2020) and shall discuss its possible applications to get average values of divisor functions.

Speaker: Gorekh Prasad
Title: Lehmer's problem under splitting conditions.
Abstract: Mahler measure of an algebraic integer α, denoted by $M(\alpha)$, is the product of all the conjugates of α that lies outside the unit circle of the complex plane. One of the longstanding open problems related to the Mahler measure is the Lehmer problem, which asks for an absolute constant $c > 1$ such that $M(\alpha) \geq c$ for any nonzero algebraic integer α which is not a root of unity. Though this problem has been verified for various classes of algebraic integers, including the class of nonreciprocal algebraic integers, the general case remains open. In this talk, after giving an overview of Lehmer's problem, we shall discuss our results on Lehmer's problem under various splitting conditions. This is a joint work with my advisor Dr. K. Senthil Kumar.

Speaker: Aditi Sevalia
Title: An induction principle for the Bombieri-Vinogradov theorem over $\mathbb{F}_q[t]$ and a variant of the Titchmarsh divisor problem.
Abstract: The Bombieri-Vinogradov theorem establishes that the primes are equidistributed in arithmetic progressions "on average" for moduli q in the range $q \leq x^{1/2-\epsilon}$ for any $\epsilon > 0$. Let $\mathbb{F}_q[t]$ be the polynomial ring over the finite field $\mathbb{F}_q$. For arithmetic functions $\psi_1; \psi_2 : \mathbb{F}_q[t] \to \mathbb{C}$, we establish that if a Bombieri-Vinogradov type equidistribution result holds for ψ_1 and ψ_2, then it also holds for their Dirichlet convolution $\psi_1 * \psi_2$. As an application, we obtain an asymptotic for the average behavior of the divisor function over shifted products of two primes in $\mathbb{F}_q[t]$. This is joint work with Sampa Dey.

Speaker: Dilip Kumar Sahoo
Title: Holomorphicity of the desingularized multiple zeta functions.

Abstract: We give an elementary proof of the holomorphicity of the desingu- larized multiple zeta functions of Furusho et al.(Amer. J. Math. 139 (2017), no. 1, 147-173) by means of a recurrence formula. Using this recurrence formula, the values of desingularized multiple zeta functions at non-positive integer points can be obtained very easily.

Seeker: V. K. Adersh
Title: On newforms and Saito-Kurokawa lifts.
Abstract: We derive the Saito-Kurokawa isomorphism on the space of new- forms for Maaß spezialschar of degree 2, weight k, level M, where $32|M$ and primitive character χ modulo$M$ with $\chi(-1) = (-1)k$ and $\chi 2$ is primitive modulo $\frac{M}{2}$. We first develop the corresponding theory of newforms for respective spaces of half-integral weight modular forms, Jacobi forms and then for Maass forms and as a consequence we get the Saito-Kurokawa isomorphism. (Joint work with Dr. M Manickam and Sreejith M M.)

Speaker - Prashant Tiwari
Title: On sign changes of primitive Fourier coefficients of Siegel cusp forms.
Abstract: In this talk, we will discuss about quantitative results for sign changes in certain sub-sequences of primitive Fourier coefficients of a non-zero Siegel cusp form of arbitrary degree over congruence subgroups. As a corollary of our result, we get sign changes of diagonal Fourier coefficients of degree two Siegel cusp forms as well. In the course of our proofs, we prove the non-vanishing of certain type of Fourier-Jacobi coefficients of a Siegel cusp form and all theta components of certain Jacobi cusp forms of arbitrary degree over congruence subgroups.

Speaker: Shivani Goel
Title: Higher convolutions of Ramanujan sums and the Hardy-Littlewood prime tuples conjec ture.
Abstract: In this talk, we introduce triple convolution Ramanujan sums and using this give a heuristic derivation of the Hardy-Littlewood conjecture on prime 3-tuples without using the circle method.

Speaker: Ramakrishnan Balakrishnan
Title: Some remarks on the Shimura correspondence.
Abstract: The correspondence (developed by Shimura) between the spaces of modular forms of half-integral weight and integral weight

plays a major role in the theory of automorphic forms. A special property of this correspondence on a class of functions was observed by H. Cohen and A. Selberg. Later the observation of Selberg was extended to a wider class of functions by several authors. In this talk, we give a connection between the observations of Cohen and Selberg. We also describe our recent work on extending this connection to a wider class. This is an ongoing joint work with B. Sahu, M. Pandey and A. K. Singh.

Speaker: M. Manikam
Title: Simultaneous non vanishing of central values of the twisted L-functions of two normalised newforms of weight $2k$ and level p, an odd prime.
Abstract: Aim of the talk to show the number of the normalised newforms g for which $L(f, D, k)L(g, D, k)$ never zero for a given normalised newform f (f,g are of weight $2k$,level$p$), for some fundamental discriminant D is grater than greater than $p^{1/2-t}$, for any $t > 0$. The involved constant depends on k, t.

Speaker: J. Sen Gupta
Title: TBA.

Speaker: Karam Deo Shankhadhar
Title: Mock modular forms whose shadows are Eisenstein series and theta powers.
Abstract: We discuss the mock modular forms whose shadows are Eisenstein series of integral and half-integral weights. As an application, we construct mock modular forms whose shadows are powers of the theta series. These mock modular forms together with their shadows give us harmonic weak Maass forms of polynomial growth. We discuss the space of the later objects and formulate an analogue of Weil's converse theorem for them. This talk is based on joint works with Ajit Bhand and Ranveer Kumar Singh.

Speaker: Abhas Kumar Jha
Title: L- functions associated with Jacobi forms.
Abstract: In this talk, we shall associate certain L- functions to Jacobi forms of half integral weight and discuss its analytic properties.

Speaker: Akshaa Vatwani
Title: Limitations to equidistribution in arithmetic progressions.
Abstract: It is well known that the prime numbers are equidistributed in arithmetic progressions. Such a phenomenon is also observed more generally for a class of arithmetic functions. A key result in this context is the Bombieri-Vinogradov theorem which establishes that the primes are equidistributed in arithmetic progressions "on average" for moduli q in the range $q \leq x^{1/2-\epsilon}$ for any $\epsilon > 0$. In 1989, building on an idea of Maier, Friedlander and Granville showed that such equidistribution results fail if the range of the moduli q is extended to $q \leq x/(\log x)^B$ for any $B > 1$. We discuss variants of this result and give some applications. This is joint work with Aditi Savalia.

Speaker: Saurabh Singh
Title: Integer points on quadrics with automorphic weights.
Abstract We are interested in counting rational points on quadratics with automorphic weights. We consider the sum

$$S = \sum_{\substack{m_i \sim X \\ F(\mathbf{m})=0}} A(m_1),$$

where F is the diagonal quadratic form

$$F(\mathbf{m}) := F(m_1, m_2, m_3, m_4) = \sum_{1 \leq i \leq 4} c_i m_i^2,$$

and $A(\cdot)$ are the Fourier coefficients of a $SL(2, \mathbb{Z})$ or a $SL(3, \mathbb{Z})$ form. We will prove a non-trivial upper bound for the sum S.

Speaker: Arvind Kumar
Title: Ramanujan-style congruences.
Abstract: Ramanujan in 1916 proved the notable congruence $\tau(n) \equiv \sigma_{11}(n) (\bmod\ 691)$ between the two important arithmetic functions: namely, the Ramanujan tau function and the 11-th divisor function. In other words, this says that there is a congruence between the unique cuspidal Hecke eigenform and the Eisenstein series of weight 12 and level 1 modulo the prime 691. The existence of such congruences opened the door for many modern developments in the theory of modular forms.
For newforms of prime level, some partial results about the existence of such congruences are known. In this talk, we discuss and refine some of those results. The main ingredients to establish our results are some classical theorems from the theory of Galois representations attached to newforms.

This talk is based on a joint work with M. Kumari, P. Moree and S. K. Singh.

Speaker: Prof. S.D Adhikari
Title: Some algebraic methods in zero-sum problems of additive combinatorics.
Abstract: Going through some results in additive combinatorics, we shall see various elementary algebraic techniques.

Speaker : Prof. A. Sankaranarayanan
Title : On the average behavior of coefficients related to triple product L-functions.
Abstract : In this talk, we discuss the asymptotic behaviour of the two sums

$$\sum_{n \leq x} \lambda_{f \otimes f \otimes f}(n)^2$$

and

$$\sum_{n \leq x} \lambda_{\mathrm{sym}^2 f \otimes f}(n)^2$$

where for instance, $\lambda_{f \otimes f \otimes f}(n)$ is the nth Fourier coefficient of the triple product L-function of a primitive holomorphic cusp form f of the full modular group $SL(2, \mathbb{Z})$. This is a joint work with Mr. K. Venkatasubbareddy.

Speaker: Anup Dixit.
Title: Lehmer's conjecture for certain infinite extensions
Abstract: For an algebraic number x, let $h(x)$ denote the logarithmic Weil height of x. In 1933, Lehmer posed the question of whether there is an absolute positive constant c, such that $h(x)deg(x) > c$, where $deg(x)$ is the degree of the minimal polynomial of x. This is known as Lehmer's conjecture and is one of the central problems in Diophantine geometry. Although the conjecture still remains open, significant progress has been made in recent times. In this talk, we will establish this conjecture for certain infinite extensions of Q and address a related question of Bombieri and Zannier. This is joint work with Sushant Kala.

Speaker: R. Thangadurai.
Title: TBA.

Speaker: Stephan Baier
Title: Solutions of $x_1^2 + x_2^2 - x_3^2 = n^2$ with small x_3
Abstract: Friedlander and Iwaniec investigated integral solutions (x_1, x_2, x_3) of the equation $x_1^2 + x_2^2 - x_3^2 = D$, where D is square-free and satisfies the congruence condition $D \equiv 5 \bmod 8$. They obtained an asymptotic formula for solutions with $x_3 \asymp M$, where M is much smaller than $\sqrt{D}$. To be precise, their condition is $M \geq D^{1/2-1/1332}$. Their analysis led them to averages of certain Weyl sums. The condition of D being square-free is essential in their work. We investigate the "opposite" case when $D = n^2$ is a square of an odd integer n. This case is different in nature and leads to sums of Kloosterman sums. We obtain an asymptotic formula for solutions with $x_3 \asymp M$, where $M \geq D^{1/2-1/16+\varepsilon}$.

Speaker: Kotyada Srinivas
Title: TBA

Speaker: Shanta Laishram
Title: On the stability of certain higher degree polynomials.
Abstract: We study the stability of $f(z) = z^d + \frac{1}{c}$ for $d \geq 3$ and c a non-zero integer. We show that whenever $f(z)$ is irreducible over rationals, all its iterates are irreducible over rationals, that is, f(z) is stable over rationals for infinitely many values of d. Thisis a joint work with R. Sarma and H. Sharma.

Speaker: Kasi Viswanadham
Title: Discrepancy of generalized polynomials
Abstract: In this talk we present an upper bound for the discrepancy of generalized monomials at primes. This is a joint work with CG Karthick Babu and Anirban Mukhopadhyay.

Speaker: Siddhi Pathak
Title: TBA

Speaker: Kummari Mallesham

Title: Second moment of derivatives of $GL(2)$ L-functions over quadratic twist.

Abstract: Let f be an Hecke eigenform of even weight k for the group $\Gamma_0(q)$. In this talk I will explain how to get, unconditionally, an asymptotic formula

$$\sum_{\substack{(d,2q)=1 \\ \omega(f\otimes\chi_{8d})=-1}}^{\star} |L'(1/2, f\times\chi_{8d})|^2 J\left(\frac{8d}{X}\right) = c_f\,\tilde{J}(1)\,X\log^3 X + O\left(X(\log X)^{\frac{5}{2}+\epsilon}\right),$$

where c_f is the constant depends only on f. This asymptotic is known, conditionally on GRH, previously by the work of Ian Petrow. This is a joint work with Sumit, Prahlad, and Saurabh.

Speaker: Bibekananda Maji
Title: Hardy-Littlewood-Riesz type criteria for the generalized Riemann hypothesis.
Abstract: Riesz, in 1916, proved that the Riemann hypothesis is equivalent to the bound $\sum_{n=1}^{\infty}\frac{\mu(n)}{n^2}\exp\left(-\frac{x}{n^2}\right) = O_\epsilon\left(x^{-\frac{3}{4}+\epsilon}\right)$, as $x\to\infty$, for any $\epsilon>0$. Around the same time, Hardy and Littlewood gave another equivalent criteria for the Riemann hypothesis while correcting an identity of Ramanujan. In this talk, we shall discuss a character analogue of the identity of Hardy and Littlewood and as an application, we provide an equivalent criteria for the generalized Riemann hypothesis. In particular, we obtain the bound given by Riesz as well as the bound of Hardy and Littlewood.

Speaker: Manish Pandey
Title: TBA.

Speaker: Esrafil Ali Molla
Title: Diophantine approximation with prime restriction in quadratic function fields.
Abstract: In the thirties of the last century, I. M. Vinogradov proved that the inequality $||p\alpha|| \le p^{-1/5+\varepsilon}$ has infinitely prime solutions p, where $||.||$ denotes the distance to a nearest integer. This result has subsequently been improved by many authors. In particular, Vaughan (1978) replaced the exponent 1/5 by 1/4 using his celebrated identity for the von Mangoldt function and a refinement of Fourier analytic arguments. The current record is due to Matomäki (2009) who showed the infinitude of prime solutions of the inequality $||p\alpha|| \le p^{-1/3+\varepsilon}$. This exponent 1/3 is considered the limit of the current technology. We prove function field analogues of this result for the fields $k=\mathbb{F}_q(T)$ and imaginary quadratic extensions K of k using the Riemann hypothesis for Hecke L-functions. Also we prove a function field analogue of

Vaughan's above-mentioned result (exponent $\theta = 1/4$) for real quadratic extensions of k of class number 1 using Vaughan's identity for function fields. **This is a joint work with Prof. Stephan Baier.**

Speaker: Moni Kumari
Title: Comparing Hecke eigenvalues of Siegel eigenforms.
Abstract: We will discuss various kinds of quantitative results about the comparison between the normalized Hecke eigenvalues of two distinct Siegel cuspidal Hecke eigenforms for the full symplectic group of degree 2. We also prove some simultaneous sign change results for their eigenvalues.

Speaker: Sreejith
Title: Certain plus spaces of newforms of half-integral weight.
Abstract: Let $k \geq 2$ and N be an odd and square free integer. We construct explicit Shintani maps indexed by odd fundamental discriminants $D, ((-1)^k D > 0)$ on the space of cusp forms of weight $2k$, level $4N$ with trivial character. Using this and the trace relation derived by S. Niwa (which gives isomorphism between the space of cusp forms of weight $2k$, level $4N$ and the space of cusp forms of weight $k + 1/2$, level $8N$) we prove that the subspace of newforms of weight $k + 1/2$ for $\Gamma_0(8N)$ with trivial character satisfies the plus space condition: the n-th Fourier coefficient $a_f(n) = 0$ whenever $(-1)^k n \equiv 2, 3 (\mathrm{mod}\ 4)$. Finally, we observe that the respective plus subspaces in $S_{k+1/2}\left(8N, \left(\frac{8}{\cdot}\right)\right)$ and $S_{k+1/2}\left(16N, \left(\frac{8}{\cdot}\right)\right)$ are trivial.

This is joint work with Prof. M Manickam.

Speaker: Md Ibrahim Molla
Title: Some Inverse Problems in Zero-sum Theory.
Abstract: Let G be a finite abelian group (written additively) of exponent n and $A \subset [1, n-1]$ a non-empty set. Then by the *Davenport constant of G with weight A,* which is denoted by $D_A(G)$, we mean the smallest positive integer k such that every sequence $S = (x_1, \ldots, x_k)$ over G of length at least k has a non-empty *A-weighted zero-sum subsequence,* that is, there exist a subsequence $(x_{j_1}, \ldots, x_{j_t})$ of S and $a_1, \ldots, a_t \in A$ such that $\sum_{i=1}^{t} a_i x_{j_i} = 0$, where 0 is the identity element of G.

For a particular $D_A(G)$, the *inverse problem* refers to the investigation of *all* sequences over G of length $D_A(G) - 1$, not having any non-empty A-weighted zero-sum subsequence.

The study of the inverse problems corresponding to the Davenport constant with various weight sets is a fascinating topic and has been of growing interest. In this talk, we shall discuss some results in this direction.