Course Curriculum

Doctoral Program
School of Mathematical Sciences

NATIONAL INSTITUTE OF SCIENCE EDUCATION AND RESEARCH BHUBANESWAR

SCHOOL OF MATHEMATICAL SCIENCES

COURSE STRUCTURE FOR DOCTORAL PROGRAM

After joining NISER, a Ph.D. student in the School of Mathematical Sciences has to undergo two semesters of course work in the first year. The course structure for these two semesters is described below:

Course No.	Credits	Course Name	Semester
MAT601	4	Algebra-I	I
MAT603	4	Analysis-I	I
MAT605	4	Topology-I	I
MAT607	4	Complex Analysis	II
MAT***	4	Elective-I	II
MAT***	4	Elective-II	II
MAT699	4	Project	II
MAT700	4	RM/RPE	II

Any two of the following three courses can be chosen by a student as Elective-I and Elective-II:

Course No.	Credits	Course Name	Semester
MAT602	4	Algebra-II	II
MAT604	4	Analysis-II	II
MAT606	4	Topology-II	II

For the Project, a student may choose a reading project (with a faculty member) or an elective from the offered electives of the school.

DETAILED SYLLABUS

Theory Courses

Course Title: Algebra-I
Course Code: MAT601
Credits: 4 Credits
Course Category: Core
Contact Hours: 56(42 Lectures + 14 Tutorials)

Outcome of the Course:

Students will learn basic properties of groups, rings, and modules and will be able to use these algebraic structures to solve research problems.

Course Contents:

Group Theory: Dihedral groups, Permutation groups, Group actions, Sylow's theorems, Simplicity of the alternating groups, Direct and semidirect products, Solvable groups, Nilpotent groups, Jordan Holder Theorem, Free groups.
Ring Theory: Properties of Ideals, Chinese remainder theorem, Field of fractions, Euclidean domains, Principal ideal domains, Unique factorization domains, Polynomial Rings, Irreducibility criteria, Matrix rings.
Module Theory: Examples, Quotient modules, Isomorphism theorems, Generation of modules, Free modules, Tensor products of modules, Exact sequences - Projective, Injective and Flat modules.

References:

1. D. S. Dummit and R. M. Foote, *Abstract Algebra*, John Wiley & Sons, 2004.
2. T. W. Hungerford, Algebra, *Graduate Texts in Mathematics*, 73, Springer, 1980.
3. M. Artin, *Algebra*, Prentice Hall, 1991.
4. N. Bourbaki, *Algebra*, Springer, 1989.
5. C Musili, *Introduction to Rings and Modules*, Narosa Publishing House.
6. N. S. Gopalakrishnan, *University Algebra*, New Age International.

Course Title: Analysis-I
Course Code: MAT603
Credits: 4 Credits
Course Category: Core
Contact Hours: 56(42 Lectures + 14 Tutorials)

Outcome of the Course:

Upon successful completion of the course, students will be familiar with various advanced concepts and techniques from functional analysis, measure theory and harmonic analysis (on the real line).

Course Contents:

Spaces of functions: Continuous functions on locally compact spaces, Stone-Weierstrass theorems, Ascoli-Arzela Theorem. Review of Measure theory: Sigma-algebras, measures, construction and properties of the Lebesgue measure, non-measurable sets, measurable functions and their properties. Integration: Lebesgue Integration, various limit theorems, comparison with the Riemann Integral, Functions of bounded variation and absolute continuity. Measure spaces: Signed-measures, Radon-Nikodym theorem, Product spaces, Fubini's theorem (without proof) and its applications. Lp-spaces: Holder and Minkowski inequalities, completeness, Convolutions, Approximation by smooth functions. Fourier analysis: Fourier Transform, Inverse Fourier transform, Plancherel Theorem for Real numbers.

References:

1. D. S. Bridges, *Foundations of Real and Abstract Analysis*, GTM series, Springer Verlag 1997.
2. G. B. Foland, *Real Analysis: Modern Techniques and Their Applications (2nd ed.)*, Wiley-Interscience/John Wiley Sons, Inc., 1999.
3. P. R. Halmos, *Measure Theory*, Springer-Verlag, 1974.
4. H. L. Royden, *Real Analysis*, Macmillan 1988.
5. W. Rudin, *Real and Complex Analysis*, TMH Edition, Second Edition, New-York, 1962.
6. Elliott H. Lieb and Michael Loss , *Analysis*, American Mathematical Society, 2001.

Course Title: Topology-I
Course Code: MAT605
Credits: 4 Credits
Course Category: Core
Contact Hours: 56(42 Lectures + 14 Tutorials)

Outcome of the Course:

Upon successful completion of the course, students will aware of various properties of topological space and various properties of functions on topological spaces. The students also learn continuous maps between topological spaces, product topology, Quotient spaces, Connectedness, Compactness, Path connected spaces, separation axioms, Tychonoff spaces, Urysohn's lemma and metrization theorem. Furthermore, the student will learn various prop-erties of functions for several variables.

Course Contents:

Topological spaces, Continuous maps between topological spaces, Product topology, Quotient spaces, Connectedness, Compactness, Path connected spaces, Separation axioms, Tychonoff spaces, Urysohn's lemma and Metrization theorem.
Differentiable functions on R^n, Jacobian criteria, Taylor's theorem, Inverse function theorem, Implicit function theorem, Maxima-minima, Lagrange multiplier.

References:

1. Armstrong, *Basic Topology,* Springer, 1983.
2. Munkres, *Topology,* Pearson Education, 2005.
3. J. Dugundji, Topology.
4. J. J. Duistermaat, J. A. C. Kolk: *Multidimensional Real Analysis I: Differentiation.*
5. K. Janich, *Topology*, Springer.
6. John L Kelley: *General Topology* (free download: https://archiveorg/details/GeneralTopology)
7. F. Simmons: *Introduction to Topology and Modern Analysis.*
8. S.Kumaresan: *A Course in Differential Geometry and Lie Groups*, TRIM series.
9. T. M. Apostol: *Calculus: Multi-Variable Calculus and Linear Algebra With Applications to Differential Equations And Probability- Vol 2*.

Course Title: Complex Analysis
Course Code: MAT607
Credits: 4 Credits
Course Category: Core
Contact Hours: 56(42 Lectures + 14 Tutorials)

Outcome of the Course:

Upon successful completion of the course, students will learn some important theorems in complex analysis such as Riemann mapping theorem, Weierstrass factorization theorem, Runges theorem, Hardamard factorization theorem, Little Picards theorem and Great Picards theorem. They will also learn some basic techniques of harmonic functions and characterization of Dirichlet Region. These results are very useful in many branches of mathematics such as Number Theory, Differential Geometry, Operator theory, Partial Differential Equations etc.

Course Contents:

Review of basic Complex Analysis: Cauchy-Riemann equations, Cauchy's theorem and estimates, power series expansions, maximum modulus principle, Classification of singularities and calculus of residues; Normal families, Arzela-Ascoli theorem, Riemann mapping theorem; Weierstrass factorization theorem, Runges theorem, Mittag-Lefflers theorem; Hadamard factorization theorem, Analytic Continuation, Gamma and Zeta functions.

References:

1. L. V. Ahlfors, *Complex Analysis*, Tata McGraw-Hill, 2013.

2. J. B. Conway, *Functions of one complex variable*, Second edition. Graduate Texts in Mathematics, 11. Springer-Verlag, New York-Berlin, 1978.

3. R. Narasimhan and Y. Nievergelt, *Complex analysis in one variable*, Second edition. Birkhuser Boston, Inc., Boston, MA, 2001.

4. W. Rudin, *Real and Complex Analysis*, Tata McGraw-Hill, 2013.

5. Wolfgang Fischer, Ingo Lieb, *A Course in Complex Analysis: From Basic Results to Advanced Topics*, Springer, 2012

6. Eberhard Freitag, Rolf Busam, *Complex Analysis*, Springer, 2005.

7. Stein and Shakarchi, *Complex Analysis*, Princeton University Press, 2003.

8. T. Gamelin, *Complex Analysis*, Springer, 2000.

Course Title: Algebra-II
Course Code: MAT602
Credits: 4 Credits
Course Category: Elective
Contact Hours: 56(42 Lectures + 14 Tutorials)

Outcome of the Course:

Students will learn basic properties of fields and Galois theory and will be able to use these results to solve other mathematical problems.

Course Contents:

Linear Algebra: Matrix of a Linear transformation, dual vector spaces, determinants, Tensor algebras, Symmetric algebras, Exterior algebras,

Modules over PIDs: Basic theory, Structure theorem for finitely generated abelian groups, Rational and Jordan canonical forms.

Field Theory: Algebraic extensions, Splitting fields, Algebraic closures, Separable and Inseparable extensions, Cyclotomic polynomials and extensions, Galois extensions, Fundamental Theorem of Galois theory, Finite fields, Composite extensions, Simple extensions, Cyclotomic extensions and Abelian extensions over rational field, Galois groups of polynomials, Fundamental theorem of algebra, Solvable and Radical extensions, Computation of Galois groups over rational field.

References:

1. D. S. Dummit and R. M. Foote, *Abstract Algebra*, John Wiley & Sons, 2004.
2. T. W. Hungerford, *Algebra*, Graduate Texts in Mathematics, 73, Springer, 1980.
3. M. Artin, *Algebra*, Prentice Hall, 1991.
4. T. T. Moh: *Algebra*, World Scientific, 1992
5. N. Bourbaki, *Algebra*, Springer, 1989.

Course Title: Analysis-II
Course Code: MAT604
Credits: 4 Credits
Course Category: Elective
Contact Hours: 56(42 Lectures + 14 Tutorials)

Outcome of the Course:

Upon successful completion of the course, students will aware of various properties of norm linear vector spaces and topological vector spaces. They will also learn various properties of linear transformations defined on these norm liner spaces and topological vector spaces.

Course Contents:

Banach spaces: Review of Banach spaces, Hahn-Banach Theorem and its applications, Baire Category theorem and its applications like Closed graph theorem, Open mapping theorem. Topological Vector spaces: Weak and weak* topologies, locally convex topological vector spaces. Hilbert spaces: Review of Hilbert spaces and operator Theory, Compact operators, Schauder's theorem on the spectral theory of compact operators. Banach algebras: Elementary properties, Resolvent and spectrum, Spectral radius formula, Ideals and homo- morphisms, Gelfand transforms, Gelfand theorem for commutative Banach algebras.

References:

1. D. S. Bridges, *Foundations of Real and Abstract Analysis*, GTM series, Springer Verlag 1997.
2. G. B. Foland, *Real Analysis: Modern Techniques and Their Applications* (2nd ed.), Wiley-Interscience/John Wiley Sons, Inc., 1999.
3. G. K. Pederson, *Analysis NOW*, GTM series, Springer-Verlag, 1991.
4. W. Rudin, *Real and Complex Analysis*, TMH Edition, Second Edition, New-York, 1962.
5. W. Rudin, *Functional Analysis*, TMH Edition, 1974.
6. K. Yosida, *Functional Analysis*, Springer-Verlag 1968.

Course Title: Topology-II
Course Code: MAT606
Credits: 4 Credits
Course Category: Elective
Contact Hours: 56(42 Lectures + 14 Tutorials)

Outcome of the Course:

Upon successful completion of the course, students will learn covering spaces, homotopy theory and several concepts of topological invariants, namely, fundamental groups, homology groups.

Course Contents:

Homotopy Theory: Fundamental groups and its functorial properties, examples, Van-Kampen Theorem, Computation of fundamental group of S^1.

Covering spaces: Covering spaces, Computation of fundamental groups using coverings. The classification of covering spaces. Deck transformations.

Simply connected spaces: Simply connected spaces-Universal covering spaces of locally simply connected and pathwise connected spaces. Universal covering group of connected subgroups of General Linear groups.

Homology groups: Affine spaces, simplexes and chains - Homology groups - Properties of Homology groups. Chain Complexes, Relation Between one dimensional Homotopy and Homology groups. Computation of Homology groups S^n, Brouwer's fixed point theorem.

References:

1. M. A. Armstrong, *Basic Topology*, Springer, 1983
2. M. J. Greenberg & J. R. Harper, *Algebraic Topology*: A First Course, Addition Wesley, 1984.
3. J. R. Munkres, *Topology*, Pearson Education, 2005. 1974