Instructions for PhD. written test

Depending on the number of students appearing in the written test, a cut off mark may be decided by the committee to short list the candidates to appear in the interview.

Syllabus for Written Test

Algebra:

Linear Algebra: Vector spaces, Linear transformations and their matrix representations, Determinants and their properties, Eigenvalues and eigenvectors, Characteristic polynomial and minimal polynomial, Diagonalization, Canonical forms.
Groups: Normal subgroups, Isomorphism theorems, permutation groups, group actions, Sylows theorem, classification of finite abelian groups.
Rings: Ideals, Isomorphism theorems, Field of fractions, Euclidean Domains, Principal Ideal Domains, Unique Factorization Domains, Polynomial rings, irreducibility criteria.
Fields: Algebraic extensions, splitting fields, separable extensions, cyclotomic polynomials, Galois extensions, Finite fields.

Analysis:

Topology of Rn, Compactness, Connectedness, Completeness. Series and sequences, Continuous and differentiable functions, Maxima and minima, Riemann integration.
Sequence and series of functions, Arzela-Ascoli theorem.
Qualitative theory of ODEs, 2x2 linear systems of ODEs.
Lebesgue measure, Basic properties of Lebesgue integral, Lp Spaces, convergence in measure, Monotone and Dominated convergence theorems, Fatou's lemma.
Banach and Hilbert spaces, Dual spaces, Compact operators, Linear operators, Hahn-Banach Theorem, Open mapping theorem,
Closed graph theorem and Uniform boundedness principle, Fourier series.
Holomorphic functions, Contour integration, Cauchy's theorem and Cauchy integral formula, Liouville's theorem, Maximum modulus principle,
Singularities, Laurent series, Theory of residues, Argument principle, Rouche's theorem, The Open Mapping theorem, Schwarz lemma.

Discrete Mathematics:

Pigeonhole principle, Counting principles, Binomial coefficients, Principles of inclusion and exclusion, recurrence relations, generating functions, Partition numbers, Partially ordered sets, Lattices, Boolean algebra.
Graphs, graph isomorphisms, degree sequence, trees, bipartite graphs, Hamilton cycles, Euler tours, directed graphs, matching, Tuttes theorem, connectivity, Mengers

theorem, planar graphs, vertex and edge colouring of graphs, matrices associated with graphs.
Divisibility, Primes, Fundamental theorem of arithmetic, Congruences, Chinese remainder theorem, Linear congruences, Fermats little theorem, Wilsons theorem, Euler function and its applications, Group of units, primitive roots, Quadratic residues, Jacobi symbol, Arithmetic functions, Mobius Inversion formula.

Probability:

Combinatorial probability and urn models; Conditional probability and independence; Random variables discrete and continuous; Expectations, variance and moments of random variables; Transformations of univariate random variables; Jointly distributed random variables; Conditional expectation; Generating functions; Limit theorems.

Theoretical computer Sciences:

Design and analysis of algorithms: Basic concepts, asymptotic notations, recurrence relation, sorting and searching, divide and conquer, greedy algorithm, dynamic programming, approximation algorithms (basics), randomized algorithm (basics)
Graph algorithms: graphs representation, BFS, DFS, shortest path, connectivity, cycles, trees, spanning tree, Eulerian cycle and Hamiltonian paths, independent set, coloring, chromatic number, dominating sets
Theory of Computation: finite state automata; DFA,NFA, regular expressions, regular languages, pumping lemma, context free languages and grammars, pushdown automata (PDA), Turing machine, decidability, recognizability, notions of P, NP, co-NP, reduction, NP complete problems.
Logic: propositional logic, equivalence and implications. truth tables, De Morgans law, quantifiers, inference and proofs, first order logic.
Basics of cryptology

Topology:

Topological spaces, Product topology, Compactness, Connectedness, Path connectedness, Continuous functions, Separation Axioms, Uryshons lemma, Tietze Extension Theorem, One point compactification.
Complete metric spaces, Uniform continuity, Quotient topology, Homotopy, Fundamental group: Basic definitions, Covering spaces, Fundamental group of the circle.