SYLLABUS

Integrated M.Sc. in Mathematics

(As per credits defined in HBNI Ordinances)

Applicable from the Academic Year 2024-2025

NATIONAL INSTITUTE OF SCIENCE EDUCATION AND RESEARCH

BHUBANESWAR

Course Structure for Integrated M.Sc. in Mathematics

Year/Semester	Course No.	Credits	Course Name
1/Semester I 20 Credits	BIO101	3	Biology-I
	CHE101	3	Chemistry-I
	MAT101	3	Mathematics-I
	PHY101	3	Physics-I
	BIO141	2	Biology Laboratory
	CHE141	2	Chemistry Laboratory
	HSS109	2	Technical Communication
	EPS101	2	Environmental and Earth Studies
1/Semester II 20 Credits	BIO102	3	Biology-II
	CHE102	3	Chemistry-II
	MAT102	3	Mathematics-II
	PHY102	3	Physics-II
	PHY141	2	Physics Laboratory
	CSE101	2	Introduction to Computer Programming
	HSS101	2	Introduction to Economics
	HSS133	2	Introduction to Sociology
2/Semester III 20 Credits	MAT201	4	Real Analysis
	MAT203	4	Discrete Mathematics
	MAT205	4	Linear Algebra
	MAT207	4	Number Theory
	****	2	Elective from SHSS
	****	2	Elective from SHSS
2/Semester IV 20 Credits	MAT202	4	Group Theory
	MAT204	4	Metric Spaces
	MAT206	4	Probability Theory
	MAT208	4	Graph Theory
	****	2	Elective from SHSS
	****	2	Elective from SHSS
3/Semester V 20 Credits	MAT301	4	Rings and Modules
	MAT303	4	Differential Equations
	MAT305	4	Calculus of Several Variables
	MAT307	4	Topology
	****	4	Elective (Out-of-stream Elective 1)
3/Semester VI 20 Credits	MAT302	4	Lebesgue Integration
	MAT304	4	Field Theory
	MAT306	4	Complex Analysis
	MAT308	4	Geometry of Curves and Surfaces
	****	4	Elective (Out-of-stream Elective 2)
4/Semester VII 20 Credits	MAT401	4	Functional Analysis
	****	4	Elective (Out-of-stream Elective 3)
	****	4	Elective-I
	****	4	Elective-II
	****	4	Elective-III

4/Semester VIII 20 Credits	MAT402	4	Representations of Finite Groups
	****	4	Elective (Out-of-stream Elective 4)
	****	4	Elective-IV
	****	4	Elective-V
	****	4	Elective-VI
5/Semester IX 20 Credits	****	4	Elective-VII (In/Out of stream 5)
	****	4	Elective-VIII
	MAT597	12	Dissertation
5/Semester X 20 Credits	****	4	Elective-IX (In/Out of stream 6)
	****	4	Elective-X
	MAT598	12	Dissertation

Out of the ten elective courses (Elective-I to Elective-X) offered by the School of Mathematical Sciences, a student majoring in Mathematics should take at least eight of them.

List of Elective Courses

Course No.	Credits	Course Name
MAT451	4	Advanced Complex Analysis
MAT452	4	Advanced Functional Analysis
MAT453	4	Advanced Linear Algebra
MAT454	4	Partial Differential Equations
MAT455	4	Introduction to Stochastic Processes
MAT456	4	Algebraic Geometry
MAT457	4	Algebraic Graph Theory
MAT458	4	Algebraic Number Theory
MAT459	4	Statistics
MAT460	4	Algorithm
MAT461	4	Numerical Analysis
MAT462	4	Cryptology
MAT463	4	Finite Fields
MAT464	4	Information and Coding Theory
MAT465	4	Mathematical Logic
MAT466	4	Measure Theory
MAT467	4	Nonlinear Analysis
MAT468	4	Operator Theory
MAT469	4	Theory of Computation
MAT470	4	Abstract Harmonic Analysis
MAT471	4	Advanced Number Theory
MAT472	4	Advanced Probability
MAT473	4	Algebraic Combinatorics
MAT474	4	Foundations of Cryptography
MAT475	4	Incidence Geometry
MAT476	4	Lie Algebras
MAT477	4	Optimization Theory
MAT478	4	Advanced Partial Differential Equations
MAT479	4	Random Graphs
MAT480	4	Randomized Algorithms and Probabilistic Methods
MAT481	4	Statistical Inference-I
MAT482	4	Multivariate Statistical Analysis
MAT483	4	Introduction to Manifolds
MAT484	4	Commutative Algebra
MAT485	4	Algebraic Topology
MAT551	4	Algebraic Computation
MAT552	4	Analytic Number Theory
MAT553	4	Classical Groups
MAT554	4	Ergodic Theory
MAT555	4	Harmonic Analysis
MAT556	4	Lie Groups and Lie Algebras-I
MAT557	4	Operator Algebras

Course No.	Credits	Course Name
MAT558	4	Representations of Linear Lie Groups
MAT559	4	Harmonic Analysis on Compact Groups
MAT560	4	Modular Forms of One Variable
MAT561	4	Elliptic Curves
MAT562	4	Brownian Motion and Stochastic Calculus
MAT563	4	Differentiable Manifolds and Lie Groups
MAT564	4	Lie Groups and Lie Algebras-II
MAT565	4	Mathematical Foundations for Finance
MAT566	4	Designs and Codes
MAT567	4	Statistical Inference-II
MAT568	4	Ordered Linear Spaces
MAT569	4	Topics in H^p Spaces
MAT570	4	Introduction to Dilation Theory
MAT571	4	Arithmetic of Quadratic Forms
MAT572	4	Optimization and optimal control
MAT573	4	Introduction to Coxeter groups
MAT574	4	Introduction to Homological Algebra

Course Structure for a Minor in Mathematics

The following five courses are required to be completed to get a Minor in Mathematics.

1. MAT201 - Real Analysis (4 credits)
2. MAT202 - Group Theory (4 credits)
3. MAT205 - Linear Algebra (4 credits)
4. MAT206 - Probability Theory (4 credits)
5. MAT303 - Differential Equations (4 credits)

Program Outcome of the Course: Integrated M.Sc. in Mathematics

The Integrated M.Sc. Program in Mathematics aims to provide comprehensive training to the students so that they will be able to build a carrier in Mathematics for themselves. The program aims to train people who are oriented towards research and teaching in both basic and advanced areas of Mathematical sciences. The core courses of the program provide a basic understanding in all areas of Mathematics which will be a foundation for further study of advanced topics. The elective courses provide knowledge in specialized topics and interconnection between different areas of Mathematics. Projects/Dissertation under the guidance of the faculty members give students exposure to current research in different areas of Mathematics and imbibe effective scientific and/or technical communication in both oral and writing. After successful completion of this program students will be able to apply knowledge of Mathematics in different fields of science and technology.

Syllabus of Core Courses

Course Title	**: Mathematics-I**
Course Code	**: MAT101**
Credits	**: 3 Credits**
Course Category	**: Core**
Course Prerequisites	**: None**
Contact Hours	**: 42(including tutorials)**

Outcome of the Course:

- To learn how to prove theorems.
- To learn how to express mathematical objects.
- Understanding the construction of natural numbers and symmetry of plane figures.

Course Contents:
Method of Mathematical Proofs: Induction, Construction, Contradiction, Contrapositive. Set: Union and Intersection of sets, Distributive laws, De Morgan's Law, Finite and infinite sets. Relation: Equivalence relation and equivalence classes. Function: Injections, Surjections, Bijections, Composition of functions, Inverse function, Graph of a function. Countable and uncountable sets, Natural numbers via Peano arithmetic, Integers, Rational numbers, Real Numbers and Complex Numbers. Matrices, Determinant, Solving system of linear equations, Gauss elimination method, Linear mappings on $\mathbb{R}^2$ and $\mathbb{R}^3$, Linear transformations and Matrices. Symmetry of Plane Figures: Translations, Rotations, Reflections, Glide-reflections, Rigid motions.

References:

1. G. Polya, "How to Solve It", Princeton University Press, 2004.
2. K. B. Sinha et. al., "Understanding Mathematics", Universities Press (India), 2003.
3. M. Artin, "Algebra", Prentice-Hall of India, 2007 (Chapters 1, 4, 5).
4. J. R. Munkres, "Topology", Prentice-Hall of India, 2013 (Chapter 1).

Course Title	**: Mathematics-II**
Course Code	**: MAT102**
Credits	**: 3 Credits**
Course Category	**: Core**
Course Prerequisites	**: None**
Contact Hours	**: 42(including tutorials)**

Outcome of the Course:

- learning basic properties of real line and real valued functions.
- learning limit, continuity and derivative of real functions.

Course Contents:
Concept of ordered field, Bounds of a set, ordered completeness axiom and characterization of $\mathbb{R}$ as a complete ordered field. Archimedean property of real numbers. Modulus of real numbers, Intervals, Neighbourhood of a point. Sequences of Real Numbers: Definition and examples, Bounded sequences, Convergence of sequences, Uniqueness of limit, Algebra of limits, Monotone sequences and their convergence, Sandwich rule. Series: Definition and convergence, Telescopic series, Series with non-negative terms. Tests for convergence [without proof]: Cauchy condensation test, Comparison test, Ratio test, Root test, Absolute and conditional convergence, Alternating series and Leibnitz test. Limit of a function at a point, Sequential criterion for the limit of a function at a point. Algebra of limits, Sandwich theorem, Continuity at a point and on intervals, Algebra of continuous functions. Discontinuous functions, Types of discontinuity. Differentiability: Definition and examples, Geometric and physical interpretations, Algebra of differentiation, Chain rule, Darboux Theorem, Rolle's Theorem, Mean Value Theorems of Lagrange and Cauchy. Application of derivatives: Increasing and decreasing functions, Maxima and minima of functions. Taylor's theorem. Forms of remainder. Expansion of functions Higher order derivatives, Leibnitz rule, L'Hopital rule.

Text Book:

1. R. G. Bartle, D. R. Sherbert, "Introduction to Real Analysis", John Wiley & Sons, 1992.

References:

1. K. A. Ross, "Elementary Analysis", Undergraduate Texts in Mathematics, Springer, 2013.
2. S. K. Berberian, "A First Course in Real Analysis", Undergraduate Texts in Mathematics, Springer-Verlag, 1994.

Course Title	**: Real Analysis**
Course Code	**: MAT201**
Credits	**: 4 Credits**
Course Category	**: Core**
Course Prerequisites	**: MAT102**
Contact Hours	**: 56(including tutorials)**

Outcome of the Course:

- Knowledge on continuity, differentiability and Riemann integration theory.
- Understanding sequence and series and it's application to numerical analysis.

Course Contents:
Countability of a set, Countability of rational numbers, Uncountability of real numbers. Limit point of a set, Bolzano-Weirstrass theorem, Open sets, Closed sets, Dense sets. Subsequence, Limit superior and limit inferior of a sequence, Cauchy criterion for convergence of a sequence, Monotone subsequence. Tests of convergence of series, Abel's and Dirichlet's tests for series, Riemann rearrangement theorem. Continuous functions on closed and bounded intervals, Intermediate value theorem, Monotone functions, Continuous monotone functions and their invertibility, Discontinuity of monotone functions. Uniform continuity, Equivalence of continuity and uniform continuity on closed and bounded intervals, Lipschitz condition, Other sufficient condition for uniform continuity. Riemann Integration: Darboux's integral, Riemann sums and their properties, Algebra of Riemann integrable functions, Class of Riemann integrable functions, Mean value theorem, Fundamental theorems of calculus, Change of variable formula (statement only), Riemann-Stieltjes integration (definition). Taylor's theorem and Taylor's series, Elementary functions. Improper integral, Beta and Gamma functions.

Text Books:

1. R. G. Bartle, D. R. Sherbert, "Introduction to Real Analysis", John Wiley & Sons, 1992.
2. K. A. Ross, "Elementary Analysis", Undergraduate Texts in Mathematics, Springer, 2013.

References:

1. T. M. Apostol, "Calculus Vol. I", Wiley-India edition, 2009.
2. S. K. Berberian, "A First Course in Real Analysis", Undergraduate Texts in Mathematics, Springer-Verlag, 1994.

Course Title	**: Group Theory**
Course Code	**: MAT202**
Credits	**: 4 Credits**
Course Category	**: Core**
Course Prerequisites	**: MAT101**
Contact Hours	**: 56(including tutorials)**

Outcome of the Course:

- Understanding the notion of symmetries in the language of groups.
- Learning various properties of groups and subgroups.

Course Contents:

Groups, subgroups, normal subgroups, quotient groups, homomorphisms, isomorphism theorems, automorphisms, permutation groups, group actions, Sylow's theorem, direct products, finite abelian groups, semi-direct products, free groups.

Text Book:

1. D. S. Dummit, R. M. Foote, "Abstract Algebra", Wiley-India edition, 2013.

References:

1. I. N. Herstein, "Topics in Algebra", Wiley-India edition, 2013.
2. M. Artin, "Algebra", Prentice-Hall of India, 2007.

Course Title	**: Discrete Mathematics**
Course Code	**: MAT203**
Credits	**: 4 Credits**
Course Category	**: Core**
Course Prerequisites	**: MAT101**
Contact Hours	**: 56(including tutorials)**

Outcome of the Course:

Learning different combinatorial techniques to solve many counting problems and understanding some mathematical structures

Course Contents:

Pigeonhole principle, Counting principles, Binomial coefficients, Principles of inclusion and exclusion, recurrence relations, generating functions, Catalan numbers, Stirling numbers, Partition numbers, Schröder numbers, Block designs, Latin squares, Partially ordered sets, Lattices, Boolean algebra.

Text Books:

1. R. A. Brualdi, "Introductory Combinatorics", Pearson Prentice Hall, 2010.
2. J. P. Tremblay, R. Manohar, "Discrete Mathematical Structures with Application to Computer Science", Tata McGraw-Hill Edition, 2008.

References:

1. J. H. van Lint, R. M. Wilson, "A Course in Combinatorics", Cambridge University Press, 2001.
2. I. Anderson, "A First Course in Discrete Mathematics", Springer Undergraduate Mathematics Series, 2001.
3. R. P. Stanley, "Enumerative Combinatorics Vol. 1", Cambridge Studies in Advanced Mathematics, 49, Cambridge University Press, 2012.

Course Title	**: Metric Spaces**
Course Code	**: MAT204**
Credits	**: 4 Credits**
Course Category	**: Core**
Course Prerequisites	**: MAT201**
Contact Hours	**: 56(including tutorials)**

Outcome of the Course:

Understanding the generalisation of euclidean distance on arbitrary sets and various properties of functions defined on them.

Course Contents:

Metric spaces, open balls and open sets, limit and cluster points, closed sets, dense sets, complete metric spaces, completion of a metric space, Continuity, uniform continuity, Banach contraction principle, Compactness, Connectedness, pathconnected sets. Sequences of functions, Pointwise convergence and uniform convergence, Arzela-Ascoli Theorem, Weierstrass Approximation Theorem, power series, radius of convergence, uniform convergence and Riemann integration, uniform convergence and differentiation, Stone-Weierstrass theorem for compact metric spaces.

Text Books:

1. G. F. Simmons, “Introduction to Topology and Modern Analysis”, Tata McGraw-Hill, 2013.
2. S. Kumaresan, “Topology of Metric Spaces”, Narosa Publishing House, 2005.

References:

1. R. R. Goldberg, “Methods of Real Analysis”, John Wiley & Sons, 1976.
2. G. B. Folland, “Real Analysis”, Wiley-Interscience Publication, John Wiley & Sons, 1999.

Course Title	**: Linear Algebra**
Course Code	**: MAT205**
Credits	**: 4 Credits**
Course Category	**: Core**
Course Prerequisites	**: MAT101**
Contact Hours	**: 56(including tutorials)**

Outcome of the Course:

- Understanding the relation between linear transformations and matrices
- Learning various fundamental results of matrices, namely, diagonalisation, triangulation and primary decomposition theorem.

Course Contents:

System of Linear Equations, Matrices and elementary row operations, Row-reduced echelon form of matrices, Vector spaces, subspaces, quotient spaces, bases and dimension, direct sums, Linear transformations and their matrix representations, Dual vector spaces, transpose of a linear transformation, Polynomial rings (over a field), Determinants and their properties, Eigenvalues and eigenvectors, Characteristic polynomial and minimal polynomial, Triangulation and Diagonalization, Simultaneous Triangulation and diagonalization, Direct-sum decompositions, Primary decomposition theorem.

Text Book:

1. K. Hoffman, R. Kunze, "Linear Algebra", Prentice-Hall of India, 2012.

References:

1. S. H. Friedberg, A. J. Insel, L. E. Spence, "Linear Algebra", Prentice Hall, 1997.
2. A. Ramachandra Rao, P. Bhimasankaram, "Linear Algebra", Texts and Readings in Mathematics, 19. Hindustan Book Agency, New Delhi, 2000.
3. M. Artin, "Algebra", Prentice-Hall of India, 2007.

Course Title	**: Probability Theory**
Course Code	**: MAT206**
Credits	**: 4 Credits**
Course Category	**: Core**
Course Prerequisites	**: MAT102**
Contact Hours	**: 56(including tutorials)**

Outcome of the Course:
Understanding the basic theory of probability starting from axiomatic definition of probability up to limit theorems of probability.

Course Contents:
Combinatorial probability and urn models; Conditional probability and independence; Random variables – discrete and continuous; Expectations, variance and moments of random variables; Transformations of univariate random variables; Jointly distributed random variables; Conditional expectation; Generating functions; Limit theorems; Simple symmetric random walk.

Text Books:

1. S. Ross, "A First Course in Probability", Pearson Education, 2012.
2. D. Stirzaker, "Elementary Probability", Cambridge University Press, Cambridge, 2003.

References:

1. K. L. Chung, F. AitSahlia, "Elementary Probability Theory", Undergraduate Texts in Mathematics. Springer-Verlag, 2003.
2. P. G. Hoel, S. C. Port, C. J. Stone, "Introduction to Probability Theory", The Houghton Mifflin Series in Statistics. Houghton Mifflin Co., 1971.
3. W. Feller, "An Introduction to Probability Theory and its Applications Vol. 1 and Vol. 2", John Wiley & Sons, 1968, 1971.

Course Title	**: Number Theory**
Course Code	**: MAT207**
Credits	**: 4 Credits**
Course Category	**: Core**
Course Prerequisites	**: MAT101**
Contact Hours	**: 56(including tutorials)**

Outcome of the Course:
Learning the elementary properties of rings of integers including divisibility, congruences, continued fractions and Gauss reciprocity laws.

Course Contents:
Divisibility, Primes, Fundamental theorem of arithmetic, Congruences, Chinese remainder theorem, Linear congruences, Congruences with prime-power modulus, Fermat's little theorem, Wilson's theorem, Euler function and its applications, Group of units, primitive roots, Quadratic residues, Jacobi symbol, Binary quadratic form, Arithmetic functions, Möbius Inversion formula, Dirichlet product, Sum of squares, Continued fractions and rational approximations.

Text Book:

1. I. Niven, H. S. Zuckerman, H. L. Montgomery, "An Introduction to the Theory of Numbers", Wiley-India Edition, 2008.

References:

1. T. M. Apostol, "Introduction to Analytic Number Theory", Springer International Student Edition, 2000.
2. G. A. Jones, J. M. Jones, "Elementary Number Theory", Springer Undergraduate Mathematics Series. Springer-Verlag, 1998.

Course Title	**: Graph Theory**
Course Code	**: MAT208**
Credits	**: 4 Credits**
Course Category	**: Core**
Course Prerequisites	**: MAT101**
Contact Hours	**: 56(including tutorials)**

Outcome of the Course:
Understanding the fundamentals of graph theory and learning the structure of graphs and techniques used to analyze different problems

Course Contents:
Graphs, subgraphs, graph isomorphisms, degree sequence, paths, cycles, trees, bipartite graphs, Hamilton cycles, Euler tours, directed graphs, matching, Tutte's theorem, connectivity, Menger's theorem, planar graphs, Kuratowski's theorem, vertex and edge colouring of graphs, network flows, max-flow min-cut theorem, Ramsey theory for graphs, matrices associated with graphs.

Text Book:

1. R. Diestel, "Graph Theory", Graduate Texts in Mathematics, 173. Springer, 2010.

References:

1. B. Bollobás, "Modern Graph Theory", Graduate Texts in Mathematics, 184. Springer-Verlag, 1998.
2. F. Harary, "Graph Theory", Addison-Wesley Publishing Co., 1969.
3. J. A. Bondy, U. S. R. Murty, "Graph Theory", Graduate Texts in Mathematics, 244. Springer, 2008.

Course Title	**: Rings and Modules**
Course Code	**: MAT301**
Credits	**: 4 Credits**
Course Category	**: Core**
Course Prerequisites	**: MAT101**
Contact Hours	**: 56(including tutorials)**

Outcome of the Course:

Understanding the structure and various properties of rings and modules, structure of finitely generated modules over PID.

Course Contents:

Rings, ideals, quotient rings, ring homomorphisms, isomorphism theorems, prime ideals, maximal ideals, Chinese remainder theorem, Field of fractions, Euclidean Domains, Principal Ideal Domains, Unique Factorization Domains, Polynomial rings, Gauss lemma, irreducibility criteria.
Modules, submodules, quotients modules, module homomorphisms, isomorphism theorems, generators, direct product and direct sum of modules, free modules, finitely generated modules over a PID, Structure theorem for finitely generated abelian groups, Rational form and Jordan form of a matrix, Tensor product of modules.

Text Book:

1. D. S. Dummit, R. M. Foote, “Abstract Algebra”, Wiley-India edition, 2013.

References:

1. I. N. Herstein, “Topics in Algebra”, Wiley-India edition, 2013.
2. M. Artin, “Algebra”, Prentice-Hall of India, 2007.

Course Title	**: Lebesgue Integration**
Course Code	**: MAT302**
Credits	**: 4 Credits**
Course Category	**: Core**
Course Prerequisites	**: MAT201**
Contact Hours	**: 56(including tutorials)**

Outcome of the Course:

- Understanding the concept of measures and measurable functions.
- Learning Lebesgue integration and their various properties

Course Contents:

Outer measure, measurable sets, Lebesgue measure, measurable functions, Lebesgue integral, Basic properties of Lebesgue integral, convergence in measure, differentiation and Lebesgue measure. L_p Spaces, Holder and Minkowski inequalities, Riesz-Fisher theorem, Radon-Nykodin theorem, Riesz representation theorem. Fourier series, L_2-convergence properties of Fourier series, Fourier transform and its properties.

Text Books:

1. H. L. Royden, "Real Analysis", Prentice-Hall of India, 2012.
2. G. B. Folland, "Real Analysis", Wiley-Interscience Publication, John Wiley & Sons, 1999.

References:

1. G. de Barra, "Measure Theory and Integration", New Age International, New Delhi, 2003.
2. W. Rudin, "Principles of Mathematical Analysis", Tata McGraw-Hill, 2013.

Course Title	**: Differential Equations**
Course Code	**: MAT303**
Credits	**: 4 Credits**
Course Category	**: Core**
Course Prerequisites	**: MAT201, MAT205**
Contact Hours	**: 56(including tutorials)**

Outcome of the Course:

- Learning different techniques to obtain explicit solutions of 1st order and second order ODE and its applications.
- learning general theory existence, uniqueness and continuous dependence of general ODE.
- Understanding propeties of solutions as maximum principle, asymptotic behaviour and phase portrait analysis of 2nd order equations.
- Leraning characteristics method for solving 1st order partial Differential Equations.

Course Contents:

Classifications of Differential Equations: origin and applications, family of curves, isoclines. First order equations: separation of variable, exact equation, integrating factor, Bernoulli equation, separable equation, homogeneous equations, orthogonal trajectories, Picard's existence and uniqueness theorems. Second order equations: variation of parameter, annihilator methods. Series solution: power series solutions about regular and singular points. Method of Frobenius, Bessel's equation and Legendre equations. Wronskian determinant, Phase portrait analysis for 2nd order system, comparison and maximum principles for 2nd order equations. Linear system: general properties, fundamental matrix solution, constant coefficient system, asymptotic behavior, exact and adjoint equation, oscillatory equations, Green's function. Sturm-Liouville theory. Partial Differential Equations: Classifications of PDE, method of separation of variables, characterstic method.

Text Books:

1. S. L. Ross, "Differential Equations", Wiley-India Edition, 2009.
2. E. A. Coddington, "An Introduction to Ordinary Differential Equations", Prentice-Hall of India, 2012.

References:

1. G. F. Simmons, S. G. Krantz, "Differential Equations", Tata Mcgraw-Hill Edition, 2007.
2. B. Rai, D. P. Choudhury, "A Course in Ordinary Differential Equation", Narosa Publishing House, New Delhi, 2002.
3. R. P. Agarwal, D. OŔegan, "Ordinary and Partial Differential Equations", Universitext. Springer, 2009.

Course Title	**: Field Theory**
Course Code	**: MAT304**
Credits	**: 4 Credits**
Course Category	**: Core**
Course Prerequisites	**: MAT301**
Contact Hours	**: 56(including tutorials)**

Outcome of the Course:
Learning the basic properties of fields including the fundamental theorem of Galois theory.

Course Contents:
Field extensions, algebraic extensions, Ruler and compass constructions, splitting fields, algebraic closures, separable and inseparable extensions, cyclotomic polynomials and extensions, automorphism groups and fixed fields, Galois extensions, Fundamental theorem of Galois theory, Fundamental theorem of algebra, Finite fields, Galois group of polynomials, Computations of Galois groups over rationals, Solvable groups, nilpotent groups, Solvability by radicals, Transcendental extensions.

Text Book:

1. D. S. Dummit, R. M. Foote, "Abstract Algebra", Wiley-India edition, 2013.

References:

1. I. N. Herstein, "Topics in Algebra", Wiley-India edition, 2013.
2. M. Artin, "Algebra", Prentice-Hall of India, 2007.
3. J. Rotman, "Galois Theory", Universitext, Springer-Verlag, 1998.

Course Title	**: Calculus of Several Variables**
Course Code	**: MAT305**
Credits	**: 4 Credits**
Course Category	**: Core**
Course Prerequisites	**: MAT201, MAT204**
Contact Hours	**: 56(including tutorials)**

Outcome of the Course:
Learning the notion of limits, continuity, differentiation and integration in the higher dimensional euclidean spaces.

Course Contents:
Differentiability of functions from an open subset of $\mathbb{R}^n$ to $\mathbb{R}^m$ and properties, chain rule, partial and directional derivatives, Continuously differentiable functions, Inverse function theorem, Implicit function theorem, Interchange of order of differentiation, Taylor's series, Extrema of a function, Extremum problems with constraints, Lagrange multiplier method with applications, Integration of functions of several variables, Change of variable formula (without proof) with examples of applications of the formula, spherical coordinates, Stokes theorem (without proof), Deriving Green's theorem, Gauss theorem and Classical Stokes theorem.

Text Books:

1. W. Fleming, "Functions of Several Variables", Undergraduate Texts in Mathematics. Springer-Verlag, 1977.
2. T. M. Apostol, "Calculus Vol. II", Wiley-India edition, 2009.

References:

1. W. Kaplan, "Advanced Calculus", Addison-Wesley Publishing Company, 1984.
2. T. M. Apostol, "Mathematical Analysis", Narosa Publishing House, 2013.

Course Title	**: Complex Analysis**
Course Code	**: MAT306**
Credits	**: 8 Credits**
Course Category	**: Core**
Course Prerequisites	**: MAT305**
Contact Hours	**: 56(including tutorials)**

Outcome of the Course:
Learning the concept of (complex) differentiation and integration of functions defined on the complex plane and their properties.

Course Contents:
Algebraic and geometric representation of complex numbers; elementary functions including the exponential functions and its relatives (log, cos, sin, cosh, sinh, etc.); concept of holomorphic (analytic) functions, complex derivative and the Cauchy-Riemann equations; harmonic functions. Conformal Mapping, Linear Fractional Transformations, Complex line integrals and Cauchy Integral formula, Representation of holomorpic functions in terms of power series, Morera's theorem, Cauchy estimates and Liouville's theorem, zeros of holomorphic functions, Uniform limits of holomorphic functions. Behaviour of holomorphic function near an isolated singularity, Laurent expansions, Counting zeros and poles, Argument principle, Rouche's theorem, Calculus of residues and evaluation of integrals using contour integration. The Open Mapping theorem, Maximum Modulus Principle, Schwarz Lemma.

Text Books:

1. J. B. Conway, "Functions of One Complex Variable", Narosa Publishing House, 2002.
2. R. E. Greene, S. G. Krantz, "Function Theory of One Complex Variable", American Mathematical Society, 2011.

References:

1. W. Rudin, "Real and Complex Analysis", Tata McGraw-Hill, 2013.
2. L. V. Ahlfors, "Complex Analysis", Tata McGraw-Hill, 2013.
3. T. W. Gamelin, "Complex Analysis", Undergraduate Texts in Mathematics, Springer, 2006.
4. E. M. Stein, R. Shakarchi, "Complex Analysis", Princeton University Press, 2003.

Course Title	**: Topology**
Course Code	**: MAT307**
Credits	**: 4 Credits**
Course Category	**: Core**
Course Prerequisites	**: MAT204**
Contact Hours	**: 56(including tutorials)**

Outcome of the Course:

- Learning abstract notion of topological spaces, continuous functions between topological spaces, Ursysohn Lemma, Tietze extension theorem and Tychonoff Theorem which they have learned in a particular setting of "Metric Space"'
- Lerning basic notions of fundamental groups and covering spaces and some of its applications

Course Contents:

Topological Spaces, Open and closed sets, Interior, Closure and Boundary of sets, Basis for Topology, Product Topology, Subspace Topology, Metric Topology, Compact Spaces, Locally compact spaces, Continuous functions, Open map, Homeomorphisms, Function Spaces, Separation Axioms: T1, Hausdorff, regular, normal spaces; Uryshon's lemma, Tietze Extension Theorem, One point compactification, Connected Spaces, Path Connected Spaces, Quotient Topology, Homotopic Maps, Deformation Retract, Contractible Spaces, Fundamental Group, The Brouwer fixed-point theorem.

Text Books:

1. J. R.Munkres, "Topology", Prentice-Hall of India, 2013.
2. M. A. Armstrong, "Basic Topology", Undergraduate Texts in Mathematics, Springer-Verlag, 1983.

References:

1. J. L. Kelley, "General Topology", Graduate Texts in Mathematics, No. 27. Springer-Verlag, New York-Berlin, 1975.
2. K. Jänich, "Topology", Undergraduate Texts in Mathematics. Springer-Verlag, 1984.

Course Title	**: Geometry of Curves and Surfaces**
Course Code	**: MAT308**
Credits	**: 8 Credits**
Course Category	**: Core**
Course Prerequisites	**: MAT305**
Contact Hours	**: 56(including tutorials)**

Outcome of the Course:
Knowledge on curves and surfaces, manifold and vector field some application on geometry of surfaces.

Course Contents:
Curves in two and three dimensions, Curvature and torsion for space curves, Existence theorem for space curves, Serret-Frenet formula for space curves, Jacobian theorem, Surfaces in $\mathbb{R}^3$ as 2-dimensional manifolds, Tangent spaces and derivatives of maps between manifolds, Geodesics, First fundamental form, Orientation of a surface, Second fundamental form and the Gauss map, Mean curvature, Gaussian Curvature, Differential forms, Integration on surfaces, Stokes formula, Gauss-Bonnet theorem.

Text Books:

1. M. P. Do Carmo, "Differential Geometry of Curves and Surfaces", Prentice Hall, 1976.
2. Andrew Pressley, "Elementary Differential Geometry", Springer, 2010.

References:

1. M. P. Do Carmo, "Differential Forms and Applications", Springer, 1994.
2. J. A. Thorpe, "Elementary Topics in Differential Geometry", Undergraduate texts in mathematics, Springer, 2011.

Course Title	**: Functional Analysis**
Course Code	**: MAT401**
Credits	**: 4 Credits**
Course Category	**: Core**
Course Prerequisites	**: MAT204, MAT205**
Contact Hours	**: 56(including tutorials)**

Outcome of the Course:
Understanding the concept of normed linear space and various properties of operators defined on them.

Course Contents:
Normed linear spaces and continuous linear transformations, Hahn-Banach theorem (analytic and geometric versions), Baire's theorem and its consequences – three basic principles of functional analysis (open mapping theorem, closed graph theorem and uniform boundedness principle), Computing the dual of wellknown Banach spaces, Hilbert spaces, Riesz representation theorem, Adjoint operator, Compact operators, Spectral theorem for self adjoint compact operators.

Text Books:

1. J. B. Conway, "A Course in Functional Analysis", Graduates Texts in Mathematics 96, Springer, 2006.
2. B. Bollobás, "Linear Analysis", Cambridge University Press, 1999.

References:

1. G. F. Simmons, "Introduction to Topology and Modern Analysis", Tata McGraw-Hill, 2013.

Course Title	**: Representations of Finite Groups**
Course Code	**: MAT402**
Credits	**: 4 Credits**
Course Category	**: Core**
Course Prerequisites	**: MAT202, MAT205, MAT301**
Contact Hours	**: 56(including tutorials)**

Outcome of the Course:

Learning the representation of finite groups via character theory.

Course Contents:

Group representations, Maschke's theorem and completely reducibility, Characters, Inner product of Characters, Orthogonality relations, Burnside's theorem, induced characters, Frobenius reciprocity, induced representations, Mackey's Irreducibility Criterion, Character table of some well-known groups, Representation theory of the symmetric group: partitions and tableaux, constructing the irreducible representations.

Text Book:

1. G. James, M. Liebeck, "Representations and Characters of Groups", Cambridge University Press, 2010.

References:

1. J. L. Alperin, R. B. Bell, "Groups and Representations", Graduate Texts in Mathematics 162, Springer, 1995.
2. B. Steinberg, "Representation Theory of Finite Groups", Universitext, Springer, 2012.
3. J-P. Serre, "Linear Representations of Finite Groups", Graduate Texts in Mathematics 42, Springer-Verlag, 1977.
4. B. Simon, "Representations of Finite and Compact Groups", Graduate Studies in Mathematics 10, American Mathematical Society, 2009.

Syllabus of Elective Courses

Course Title	**: Advanced Complex Analysis**
Course Code	**: MAT451**
Credits	**: 4 Credits**
Course Category	**: Elective**
Course Prerequisites	**: MAT306**
Contact Hours	**: 56(including tutorials)**

Outcome of the Course:

- Leraning some important theorems in complex analysis such as Riemann mapping theorem, Weirstrass factorization theorem, Runges theorem, Hardamard factorization theorem, Little Picards theorem and Great Picards theorem.
- Learning some basic techniques of harmonic functions and characterization of Dirichlet Region.

Course Contents:

Review of basic Complex Analysis: Cauchy-Riemann equations, Cauchy's theorem and estimates, power series expansions, maximum modulus principle, Classification of singularities and calculus of residues. Space of continuous functions, Arzela's theorem, Spaces of analytic functions, Spaces of meromorphic functions, Riemann mapping theorem, Weierstrass Factorization theorem, Runge's theorem, Simple connectedness, Mittag-Leffler's theorem, Analytic continuation, Schwarz reflection principle, Mondromy theorem, Jensen's formula, Genus and order of an entire function, Hadamard factorization theorem, Little Picard theorem, Great Picard theorem, Harmonic functions.

References:

1. L. V. Ahlfors, "Complex Analysis", Tata McGraw-Hill, 2013.
2. J. B. Conway, "Functions of One Complex Variable II", Graduate Texts in Mathematics 159, Springer-Verlag, 1996.
3. W. Rudin, "Real and Complex Analysis", Tata McGraw-Hill, 2013.
4. R. Remmert, "Theory of Complex Functions", Graduate Texts in Mathematics 122, Springer, 2008.

Course Title	**: Advanced Functional Analysis**
Course Code	**: MAT452**
Credits	**: 4 Credits**
Course Category	**: Elective**
Course Prerequisites	**: MAT401**
Contact Hours	**: 56(including tutorials)**

Outcome of the Course:

Understanding the concept of topological vector space, as a generalisation of normed linear spaces, and various properties of operators defined on them.

Course Contents:

Definition and examples of topological vector spaces (TVS) and locally convex spaces (LCS); Linear operators; Hahn-Banach Theorems for TVS/ LCS (analytic and geometric forms); Uniform boundedness principle; Open mapping theorem; Closed graph thoerem; Weak and weak* vector topologies; Bipolar theorem; dual of LCS spaces; Krein-Milman theorem for TVS; Krien-Smulyan theorem for Banach spaces; Inductive and projective limit of LCS.

References:

1. W. Rudin, "Functional Analysis", Tata McGraw-Hill, 2007.
2. A. P. Robertson, W. Robertson, "Topological Vector Spaces", Cambridge Tracts in Mathematics 53, Cambridge University Press, 1980.
3. J. B. Conway, "A Course in Functional Analysis", Graduates Texts in Mathematics 96, Springer, 2006.

Course Title	**: Advanced Linear Algebra**
Course Code	**: MAT453**
Credits	**: 4 Credits**
Course Category	**: Elective**
Course Prerequisites	**: MAT205**
Contact Hours	**: 56(including tutorials)**

Outcome of the Course:
Learning various decomposition results of matrices and their applications.

Course Contents:
Rational and Jordan canonical forms, Inner product spaces, Unitary and Normal operators, Forms on inner product spaces, Spectral theorems, Bilinear forms, Matrix decomposition theorems, Courant- Fischer minimax and related theorems, Nonnegative matrices, Perron-Frobenius theory, Generalized inverse, Matrix Norm, Perturbation of eigenvalues.

References:

1. R. A. Horn, C. R. Johnson, "Matrix Analysis", Cambridge University Press, 2010.
2. K. Hoffman, R. Kunze, "Linear Algebra", Prentice-Hall of India, 2012.
3. S. Roman, "Advanced Linear Algebra", Graduate Texts in Mathematics 135, Springer, 2008.

Course Title	**: Partial Differential Equations**
Course Code	**: MAT454**
Credits	**: 4 Credits**
Course Category	**: Elective**
Course Prerequisites	**: MAT302, MAT303, MAT305**
Contact Hours	**: 56(including tutorials)**

Outcome of the Course:

- Learning the explicit representations of solutions of four important classes of PDEs, namely, Transport equations, Heat equation, Laplace equation and wave equation for initial value problems.
- Learning the properties of solutions of these equations such as mean value property, maximum principles and regularity.
- Understanding Cauchy-Kowalevski Theorem and uniqueness theorem of Holmgreen for quasilinear equations.

Course Contents:

Classification of Partial Differential Equations, Cauchy Problem, Cauchy-Kowalevski Theorem, Lagrange-Green identity, The uniquness theorem of Holmgren, Transport equation: Initial value problem, nonhomogeneous problem. Laplace equation: Fundamental solution, Mean Value formula, properties of Harmonic functions, Green's function, Energy methods, Harnack's inequality. Heat Equation: Fundamental solution, Mean value formula, properties of solutions. Wave equation: Solution by spherical means, Nonhomogeneous problem, properties of solutions.

References:

1. L. C. Evans, "Partial Differential Equations", Graduate Studies in Mathematics 19, American Mathematical Society, 2010.
2. F. John, "Partial Differential Equations", Springer International Edition, 2009.
3. G. B. Folland, "Introduction to Partial Differential Equations", Princeton University Press, 1995.
4. S. Kesavan, "Topics in Functional Analysis and Applications", John Wiley & Sons, 1989.

Course Title	**: Introduction to Stochastic Processes**
Course Code	**: MAT455**
Credits	**: 4 Credits**
Course Category	**: Elective**
Course Prerequisites	**: MAT206**
Contact Hours	**: 56(including tutorials)**

Outcome of the Course:

Understanding the theory of discrete time and continuous time Markov chains.

Course Contents:

Discrete Markov chains with countable state space; Classification of states: recurrences, transience, periodicity. Stationary distributions, reversible chains, Several illustrations including the Gambler's Ruin problem, queuing chains, birth and death chains etc. Poisson process, continuous time Markov chain with countable state space, continuous time birth and death chains.

References:

1. P. G. Hoel, S. C. Port, C. J. Stone, "Introduction to Stochastic Processes", Houghton Mifflin Co., 1972.
2. R. Durrett, "Essentials of Stochastic Processes", Springer Texts in Statistics, Springer, 2012.
3. G. R. Grimmett, D. R. Stirzaker, "Probability and Random Processes", Oxford University Press, 2001.
4. S. M. Ross, "Stochastic Processes", Wiley Series in Probability and Statistics: Probability and Statistics, John Wiley & Sons, 1996

Course Title	**: Algebraic Geometry**
Course Code	**: MAT456**
Credits	**: 4 Credits**
Course Category	**: Elective**
Course Prerequisites	**: MAT205, MAT301**
Contact Hours	**: 56(including tutorials)**

Outcome of the Course:

- Learning the fundamentals of classical algebraic geometry.
- Learning about the theory of Riemann surfaces, divisors, line bundles, Chern Classes and the Riemann Roch Theorem.

Course Contents:

Prime ideals and primary decompositions, Ideals in polynomial rings, Hilbert Basis theorem, Noether normalisation lemma, Hilbert's Nullstellensatz, Affine and Projective varieties, Zariski Topology, Rational functions and morphisms, Elementary dimension theory, Smoothness, Curves, Divisors on curves, Bezout's theorem, Riemann-Roch for curves, Line bundles on Projective spaces.

References:

1. K. Hulek, "Elementary Algebraic Geometry", Student Mathematical Library 20, American Mathematical Society, 2003.
2. I. R. Shafarevich, "Basic Algebraic Geometry 1: Varieties in Projective Space", Springer, 2013.
3. J. Harris, "Algebraic geometry", Graduate Texts in Mathematics 133, Springer-Verlag, 1995.
4. M. Reid, "Undergraduate Algebraic Geometry", London Mathematical Society Student Texts 12, Cambridge University Press, 1988.
5. K. E. Smith et. al., "An Invitation to Algebraic Geometry", Universitext, Springer-Verlag, 2000.
6. R. Hartshorne, "Algebraic Geometry", Graduate Texts in Mathematics 52, Springer-Verlag, 1977.

Course Title	**: Algebraic Graph Theory**
Course Code	**: MAT457**
Credits	**: 4 Credits**
Course Category	**: Elective**
Course Prerequisites	**: MAT205, MAT208**
Contact Hours	**: 56(including tutorials)**

Outcome of the Course:

Learning the different algebraic techniques used in the study of the graphs

Course Contents:

Adjacency matrix of a graph and its eigenvalues, Spectral radius of graphs, Regular graphs and Line graphs, Strongly regular graphs, Cycles and Cuts, Laplacian matrix of a graph, Algebraic connectivity, Laplacian spectral radius of graphs, Distance matrix of a graph, General properties of graph automorphisms, Transitive and Arc-tranisitve graphs, Symmetric graphs.

References:

1. N. Biggs, "Algebraic Graph Theory", Cambridge University Press, 1993.
2. C. Godsil, G. Royle, "Algebraic Graph Theory", Graduate Texts in Mathematics 207, Springer-Verlag, 2001.
3. R. B. Bapat, "Graphs and Matrices", Universitext, Springer, Hindustan Book Agency, New Delhi, 2010.

Course Title	**: Algebraic Number Theory**
Course Code	**: MAT458**
Credits	**: 4 Credits**
Course Category	**: Elective**
Course Prerequisites	**: MAT207, MAT304**
Contact Hours	**: 56(including tutorials)**

Outcome of the Course:
Understanding the basic properties of number fields, computation of class numbers and zeta functions.

Course Contents:
Number Fields and Number rings, prime decomposition in number rings, Dedekind domains, Ideal class group, Galois theory applied to prime decomposition, Gauss reciprocity law, Cyclotomic fields and their ring of integers, finiteness of ideal class group, Dirichlet unit theorem, valuations and completions of number fields, Dedekind zeta function and distribution of ideal in a number ring.

References:

1. D. A. Marcus, "Number Fields", Universitext, Springer-Verlag, 1977.
2. G. J. Janusz, "Algebraic Number Fields", Graduate Studies in Mathematics 7, American Mathematical Society, 1996.
3. S. Alaca, K. S. Williams, "Introductory Algebraic Number Theory", Cambridge University Press, 2004.
4. S. Lang, "Algebraic Number Theory", Graduate Texts in Mathematics 110, Springer-Verlag, 1994.
5. A. Frohlich, M. J. Taylor, "Algebraic Number Theory", Cambridge Studies in Advanced Mathematics 27, Cambridge University Press, 1993.
6. J. Neukirch, "Algebraic Number Theory", Springer-Verlag, 1999.

Course Title	**: Statistics**
Course Code	**: MAT459**
Credits	**: 4 Credits**
Course Category	**: Elective**
Course Prerequisites	**: MAT206**
Contact Hours	**: 56(including tutorials)**

Outcome of the Course:

- Learning about the descriptive statistics of data sets including graphical representation using some statistical software.
- Understanding the basic theory of point estimation, interval estimation, hypothesis testing and linear regression.

Course Contents:

Descriptive Statistics, Graphical representation of data, Curve fittings, Simple correlation and regression, Multiple and partial correlations and regressions, Sampling, Sampling distributions, Standard error. Normal distribution and its properties, The distribution of $\overline{X}$ and S^2 in sampling from a normal distribution, Exact sampling distributions: χ^2, t, F. Theory and Methods of Estimation: Point estimation, Criteria for a good estimator, Properties of estimators: Unbiasedness, Efficiency, Consistency, Sufficiency, Robustness. A lower bound for a variance of an estimate, Method of estimation: The method of moment, Least square method, Maximum likelihood estimation and its properties, UMVU Estimator, Interval estimation. Test of Hypothesis: Elements of hypothesis testing, Unbiased test, Neyman-Pearson Theory, MP and UMP tests, Likelihood ratio and related tests, Large sample tests, Test based on χ^2, t, F.

Text Books:

1. H. J. Larson, “Introduction to Probability Theory and Statistical Inference”, John Wiley & Sons, 1982.
2. V. K. Rohatgi, “Introduction to Probability Theory and Mathematical Statistics”, John Wiley & Sons, 1976.

References:

1. I. Miller, M. Miller, “John E. Freund’s Mathematical Statistics with Applications”, Pearson, 2013.

Course Title	**: Algorithm**
Course Code	**: MAT460**
Credits	**: 4 Credits**
Course Category	**: Elective**
Course Prerequisites	**: MAT208**
Contact Hours	**: 56(including tutorials)**

Outcome of the Course:

- Learning data structure, design and analysis algorithms.
- Understanding some important algorithms like sortings, graph theoretics, polynomial related and optimization related.

Course Contents:

Algorithm analysis, asymptotic notation, probabilistic analysis; Data Structure: stack, queues, linked list, hash table, binary search tree, red-black tree; Sorting: heap sort, quick sort, sorting in linear time; Algorithm design: divide and conquer, greedy algorithms, dynamic programming; Algebraic algorithms: Winograd's and Strassen's matrix multiplication algorithm, evaluation of polynomials, DFT, FFT, efficient FFT implementation; Graph algorithms: breadth-first and depth-first search, minimum spanning trees, single-source shortest paths, all-pair shortest paths, maximum flow; NP-completeness and approximation algorithms.

References:

1. A. V. Aho, J. E. Hopcroft, J. D. Ullman, "The Design and Analysis of Computer Algorithms", Addison-Wesley Publishing Co., 1975.
2. T. H. Cormen, C. E. Leiserson, R. L. Rivest, C. Stein, "Introduction to Algorithms", MIT Press, Cambridge, 2009.
3. E. Horowitz, S. Sahni, "Fundamental of Computer Algorithms", Galgotia Publication, 1987.
4. D. E. Knuth, "The Art of Computer Programming Vol. 1, Vol. 2, Vol 3", Addison-Wesley Publishing Co., 1997, 1998, 1998.

Course Title	**: Numerical Analysis**
Course Code	**: MAT461**
Credits	**: 4 Credits**
Course Category	**: Elective**
Course Prerequisites	**: MAT201, MAT303**
Contact Hours	**: 56(including tutorials)**

Outcome of the Course:
Llearning the practical use of some important results from real analysis and linear algebra.

Course Contents:
Errors in computation: Representation and arithmetic of numbers, source of errors, error propagation, error estimation. Numerical solution of non-linear equations: Bisection method, Secant method, Newton-Raphson method, Fixed point methods, Muller's method. Interpolations: Lagrange interpolation, Newton divided differences, Hermite interpolation, Piecewise polynomial interpolation. Approximation of functions: Weierstrass and Taylor expansion, Least square approximation. Numerical Integration: Trapezoidal rule, Simpson's rule, Newton-Cotes rule, Guassian quadrature. Numerical solution of ODE: Euler's method, multi-step methods, Runge-Kutta methods, Predictor-Corrector methods. Solutions of systems of linear equations: Gauss elimination, pivoting, matrix factorization, Iterative methods – Jacobi and Gauss-Siedel methods. Matrix eigenvalue problems: power method.

Text Book:

1. K. E. Atkinson, "An Introduction to Numerical Analysis" Wiley-India Edition, 2013.

References:

1. S. D. Conte, C. De Boor, "Elementary Numerical Analysis, Tata McGraw-Hill, 2006.
2. W. H. Press et. al., "Numerical Recipes - The Art of Scientific Computing", Cambridge University Press, 2007.

Course Title	**: Cryptology**
Course Code	**: MAT462**
Credits	**: 4 Credits**
Course Category	**: Elective**
Course Prerequisites	**: MAT202, MAT207**
Contact Hours	**: 56(including tutorials)**

Outcome of the Course:

- Learning the basics of cryptography and cryptanalysis.
- Understanding the theory and design of cryptographic schemes like stream ciphers, block ciphers and public key ciphers like RSA, ElGamal, elliptic curve cryptosystem.
- Learning about data authentication, integrity and secret sharing.

Course Contents:
Overview of Cryptography and cryptanalysis, some simple cryptosystems (e.g., shift, substitution, affine, knapsack) and their cryptanalysis, classification of cryptosystems, classification of attacks; Information Theoretic Ideas: Perfect secrecy, entropy; Secret key cryptosystem: stream cipher, LFSR based stream ciphers, cryptanalysis of stream cipher (e.g., correlation attack, algebraic attacks), block cipher, DES, linear and differential cryptanalysis, AES; Public-key cryptosystem: Implementation and cryptanalysis of RSA, ElGamal public-key cryptosystem, Discrete logarithm problem, elliptic curve cryptography; Data integrity and authentication: Hash functions, message authentication code, digital signature scheme, ElGamal signature scheme; Secret sharing: Shamir's threshold scheme, general access structure and secret sharing.

References:

1. D. R. Stinson, "Cryptography: Theory And Practice", Chapman & Hall/CRC, 2006.
2. A. J. Menezes, P. C. van Oorschot, S. A. Vanstone, "Handbook of Applied Cryptography", CRC Press, 1997.

Course Title	**: Finite Fields**
Course Code	**: MAT463**
Credits	**: 4 Credits**
Course Category	**: Elective**
Course Prerequisites	**: MAT304**
Contact Hours	**: 56(including tutorials)**

Outcome of the Course:
Understanding the structures of finite fields, factorization of polynomials, some applications towards cryptography, coding theory and combinatorics.

Course Contents:
Structure of finite fields: characterization, roots of irreducible polynomials, traces, norms and bases, roots of unity, cyclotomic polynomial, representation of elements of finite fields, Wedderburn's theorem; Polynomials over finite field: order of polynomials, primitive polynomials, construction of irreducible polynomials, binomials and trinomials, factorization of polynomials over small and large finite fields, calculation of roots of polynomials; Linear recurring sequences: LFSR, characteristic polynomial, minimal polynomial, characterization of linear recurring sequences, Berlekamp-Massey algorithm; Applications of finite fields: Applications in cryptography, coding theory, finite geometry, combinatorics.

References:

1. R. Lidl, H. Neiderreiter, "Finite Fields", Cambridge university press, 2000.
2. G. L. Mullen, C. Mummert, "Finite Fields and Applications", American Mathematical Society, 2007.
3. A. J. Menezes et. al., "Applications of Finite Fields", Kluwer Academic Publishers, 1993.
4. Z-X. Wan, "Finite Fields and Galois Rings", World Scientific Publishing Co., 2012.

Course Title	**: Information and Coding Theory**
Course Code	**: MAT464**
Credits	**: 4 Credits**
Course Category	**: Elective**
Course Prerequisites	**: MAT205, MAT304**
Contact Hours	**: 56(including tutorials)**

Outcome of the Course:

- Learning how to measure information and encoding of information.
- Understanding the theory and techniques of error correcting codes like Reed-Muller codes, BCH codes, Reed-Solomon codes, Algebraic codes.

Course Contents:
Information Theory: Entropy, Huffman coding, Shannon-Fano coding, entropy of Markov process, channel and mutual information, channel capacity; Error correcting codes: Maximum likelihood decoding, nearest neighbour decoding, linear codes, generator matrix and parity-check matrix, Hamming bound, Gilbert-Varshamov bound, binary Hamming codes, Plotkin bound, nonlinear codes, Reed-Muller codes, Cyclic codes, BCH codes, Reed-Solomon codes, Algebraic codes.

References:

1. R. W. Hamming, "Coding and Information Theory", Prentice-Hall, 1986.
2. N. J. A. Sloane, F. J. MacWilliams, "Theory of Error Correcting Codes", North-Holland Mathematical Library 16, North-Holland, 2007.
3. S. Ling, C. Xing, "Coding Theory: A First Course", Cambridge University Press, 2004.
4. V. Pless, "Introduction to the Theory of Error-Correcting Codes", Wiley-Interscience Publication, John Wiley & Sons, 1998.
5. S. Lin, "An Introduction to Error-Correcting Codes", Prentice-Hall, 1970.

Course Title	**: Mathematical Logic**
Course Code	**: MAT465**
Credits	**: 4 Credits**
Course Category	**: Elective**
Course Prerequisites	**: MAT101**
Contact Hours	**: 56(including tutorials)**

Outcome of the Course:

- Learning the propositional logic and first order theory.
- Understanding the completeness and compactness theorems with Godels incompleteness theorem.

Course Contents:
Propositional Logic, Tautologies and Theorems of propositional Logic, Tautology Theorem. First Order Logic: First order languages and their structures, Proofs in a first order theory, Model of a first order theory, validity theorems, Metatheorems of a first order theory, e. g., theorems on constants, equivalence theorem, deduction and variant theorems etc. Completeness theorem, Compactness theorem, Extensions by definition of first order theories, Interpretations theorem, Recursive functions, Arithmatization of first order theories, Godels first Incompleteness theorem, Rudiments of model theory including Lowenheim-Skolem theorem and categoricity.

References:

1. J. R. Shoenfield, “Mathematical logic”, Addison-Wesley Publishing Co., 1967.
2. E. Mendelson, “Introduction to Mathematical Logic”, Chapman & Hall, 1997.

Course Title	**: Measure Theory**
Course Code	**: MAT466**
Credits	**: 4 Credits**
Course Category	**: Elective**
Course Prerequisites	**: MAT302**
Contact Hours	**: 56(including tutorials)**

Outcome of the Course:

- Learning the concept of measures and measurable functions.
- Understanding integration and their various properties

Course Contents:

$\sigma-$algebras of sets, measurable sets and measures, extension of measures, construction of Lebesgue measure, integration, convergence theorems, Radon-Nikodym theorem, product measures, Fubini's theorem, differentiation of integrals, absolutely continuous functions, L_p-spaces, Riesz representation theorem for the space $C[0,1]$.

References:

1. G. De Barra, "Measure theory and integration".
2. J. Nevue, "Mathematical foundations of the calculus of probability", Holden-Day, Inc., 1965.
3. I. K. Rana, "An introduction to measure and integration", Narosa Publishing House.
4. P. Billingsley, "Probability and measure", John Wiley & Sons, Inc., 1995.
5. W. Rudin, "Real and complex analysis", McGraw-Hill Book Co., 1987.
6. K. R. Parthasarathy, "Introduction to probability and measure", The Macmillan Co. of India, Ltd., 1977.

Course Title	**: Nonlinear Analysis**
Course Code	**: MAT467**
Credits	**: 4 Credits**
Course Category	**: Elective**
Course Prerequisites	**: MAT304, MAT401**
Contact Hours	**: 56(including tutorials)**

Outcome of the Course:

- Learning calculus in Banach Spaces, degree theory and it's application for fixed point theorems of Brouwer and Schauder.
- Learning homotopy, homotopy extension and invariance theorems and its applications.

Course Contents:

Calculus in Banach spaces, inverse and multiplicit function theorems, fixed point theorems of Brouwer, Schauder and Tychonoff, fixed point theorems for nonexpansive and set-valued maps, predegree results, compact vector fields, homotopy, homotopy extension, invariance theorems and applications.

References:

1. S. Kesavan, "Nonlinear Functional Analysis", Texts and Readings in Mathematics 28, Hindustan Book Agency, 2004.

Course Title	**: Operator Theory**
Course Code	**: MAT468**
Credits	**: 4 Credits**
Course Category	**: Elective**
Course Prerequisites	**: MAT401**
Contact Hours	**: 56(including tutorials)**

Outcome of the Course:

Learning the concepts of C*-algebra, von-Neuman algebra and toeplitz operators and the notion of index for Fredholm operators.

Course Contents:

Compact operators on Hilbert Spaces. (a) Fredholm Theory (b) Index, C*-algebras - noncommutative states and representations, Gelfand-Neumark representation theorem, Von-Neumann Algebras; Projections, Double Commutant theorem, L^{∞} functionalCalculus, Toeplitz operators.

References:

1. W. Arveson, "An invitation to C*-algebras", Graduate Texts in Mathematics, No. 39. Springer-Verlag, 1976.
2. N. Dunford and J. T. Schwartz, "Linear operators. Part II: Spectral theory. Self adjoint operators in Hilbert space", Interscience Publishers John Wiley i& Sons 1963.
3. R. V. Kadison and J. R. Ringrose, "Fundamentals of the theory of operator algebras. Vol. I. Elementary theory", Pure and Applied Mathematics, 100, Academic Press, Inc., 1983.
4. V. S. Sunder, "An invitation to von Neumann algebras", Universitext, Springer-Verlag, 1987.

Course Title	**: Theory of Computation**
Course Code	**: MAT469**
Credits	**: 4 Credits**
Course Category	**: Elective**
Course Prerequisites	**: MAT101**
Contact Hours	**: 56(including tutorials)**

Outcome of the Course:

- Learning Automata and Language theory by studying automata and context free language.
- Learning Computability theory by studying Turing machine and halting problem.
- Learning Complexity theory by studying P and NP class problems.

Course Contents:

Automata and Language Theory: Finite automata, regular expression, pumping lemma, context free grammar, context free languages, Chomsky normal form, push down automata, pumping lemma for CFL; Computability: Turing machines, Churh-Turing thesis, decidability, halting problem, reducibility, recursion theorem; Complexity: Time complexity of Turing machines, Classes P and NP, NP completeness, other time classes, the time hierarchy.

References:

1. J. E. Hopcroft, R. Motwani, J. D. Ullman, "Introduction to Automata Theory, Languages, and Computation", Addison-Wesley, 2006.
2. H. Lewis, C. H. Papadimitriou, "Elements of the Theory of Computation", Prentice-Hall, 1997.
3. M. Sipser, "Introduction to the Theory of Computation", PWS Publishing, 1997.

Course Title	**: Abstract Harmonic Analysis**
Course Code	**: MAT470**
Credits	**: 4 Credits**
Course Category	**: Elective**
Course Prerequisites	**: MAT302, MAT306, MAT401**
Contact Hours	**: 56(including tutorials)**

Outcome of the Course:

Knowledge on Haar measure, convolution structure on Lie group with emphasize to harmonic analysis on the groups Circle and real line.

Course Contents:

Topological Groups: Basic properties of topological groups, subgroups, quotient groups. Examples of various matrix groups. Connected groups. Haar measure: Discussion of Haar measure without proof on $\mathbb{R}$, $\mathbb{T}$, $\mathbb{Z}$ and simple matrix groups, Convolution, the Banach algebra $L^1(G)$ and convolution with special emphasis on $L^1(\mathbb{R})$, $L^1(\mathbb{T})$ and $L^1(\mathbb{Z})$. Basic Representation Theory: Unitary representation of groups, Examples and General properties, The representations of Group and Group algebras, C^*-algebra of a group, GNS construction, Positive definite functions, Schur's Lemma. Abelian Groups: Fourier transform and its properties, Approximate identities in $L^1(G)$, Classical Kernels on $\mathbb{R}$, The Fourier inversion Theorem, Plancherel theorem on $\mathbb{R}$, Plancherel measure on $\mathbb{R}$, $\mathbb{T}$, $\mathbb{Z}$. Dual Group of an Abelian Group: The Dual group of a locally compact abelian group, Computation of dual groups for $\mathbb{R}$, $\mathbb{T}$, $\mathbb{Z}$, Pontryagin's Duality theorem.

References:

1. G. B. Folland, "A Course in Abstract Harmonic Analysis", CRC Press, 2000.
2. H. Helson, "Harmonic Analysis", Texts and Readings in Mathematics, Hindustan Book Agency, 2010.
3. Y. Katznelson, "An Introduction to Harmonic Analysis", Cambridge University Press, 2004.
4. L. H. Loomis, "An Introduction to Abstract Harmonic Analysis", Dover Publication, 2011.
5. E. Hewitt, K. A. Ross, "Abstract Harmonic Analysis Vol. I", Springer-Verlag, 1979.
6. W. Rudin, "Real and Complex Analysis", Tata McGraw-Hill, 2013.

Course Title	**: Advanced Number Theory**
Course Code	**: MAT471**
Credits	**: 4 Credits**
Course Category	**: Elective**
Course Prerequisites	**: MAT207, MAT304, MAT306**
Contact Hours	**: 56(including tutorials)**

Outcome of the Course:
Learning p-adic numbers, quadratic forms, Dirichlet series and modular forms.

Course Contents:
Review of Finite fields, Gauss Sums and Jacobi Sums, Cubic and biquadratic reciprocity, Polynomial equations over finite fields, Theorems of Chevally and Warning, Quadratic forms over prime fields. Ring of p-adic integers, Field of p-adic numbers, completion, p-adic equations, Hensel's lemma, Hilbert symbol, Quadratic forms with p-adic coefficients. Dirichlet series: Abscissa of convergence and absolute convergence, Riemann Zeta function and Dirichlet L-functions. Dirichlet's theorem on primes in arithmetic progression. Functional equation and Euler product for L-functions. Modular Forms and the Modular Group, Eisenstein series, Zeros and poles of modular functions, Dimensions of the spaces of modular forms, The j-invariant L-function associated to modular forms, Ramanujan τ function.

References:

1. J.-P. Serre, "A Course in Arithmetic", Graduate Texts in Mathematics 7, Springer-Verlag, 1973.
2. K. Ireland, M. Rosen, "A Classical Introduction to Modern Number Theory", Graduate Texts in Mathematics 84, Springer-Verlag, 1990.
3. H. Hasse, "Number Theory", Classics in Mathematics, Springer-Verlag, 2002.
4. W. Narkiewicz, "Elementary and Analytic Theory of Algebraic Numbers", Springer Monographs in Mathematics, Springer-Verlag, 2004.
5. F. Q. Gouvêa, "p-adic Numbers", Universitext, Springer-Verlag, 1997.

Course Title	**: Advanced Probability**
Course Code	**: MAT472**
Credits	**: 4 Credits**
Course Category	**: Elective**
Course Prerequisites	**: MAT206, MAT302**
Contact Hours	**: 56(including tutorials)**

Outcome of the Course:
Learning about measure theoretic probability starting from probability spaces to theory of martingales.

Course Contents:
Probability spaces, Random Variables, Independence, Zero-One Laws, Expectation, Product spaces and Fubini's theorem, Convergence concepts, Law of large numbers, Kolmogorov three-series theorem, Levy-Cramer Continuity theorem, CLT for i.i.d. components, Infinite Products of probability measures, Kolmogorov's Consistency theorem, Conditional expectation, Discrete parameter martingales with applications.

References:

1. A. Gut, "Probability: A Graduate Course", Springer Texts in Statistics, Springer, 2013.
2. K. L. Chung, "A Course in Probability Theory", Academic Press, 2001.
3. S. I. Resnick, "A Probability Path", Birkhäuser, 1999.
4. P. Billingsley, "Probability and Measure", Wiley Series in Probability and Statistics, John Wiley & Sons, 2012.
5. J. Jacod, P. Protter, "Probability Essentials", Universitext, Springer-Verlag, 2003.

Course Title	**: Algebraic Combinatorics**
Course Code	**: MAT473**
Credits	**: 4 Credits**
Course Category	**: Elective**
Course Prerequisites	**: MAT202, MAT203**
Contact Hours	**: 56(including tutorials)**

Outcome of the Course:

Learning the use of different algebraic technique to study the combinatorial problems

Course Contents:

Catalan Matrices and Orthogonal Polynomials, Catalan Numbers and Lattice Paths, Combinatorial Interpretation of Catalan Numbers, Symmetric Polynomials and Functions, Schur Functions, Jacobi-Trudi identity, RSK Algorithm, Standard Tableaux, Young diagrams and q-binomial coefficients, Plane Partitions, Group actions on boolean algebras, Enumeration under group action, Walks in graphs, Cubes and the Radon transform, Sperner property, Matrix-Tree Theorem.

References:

1. R. P. Stanley, "Algebraic Combinatorics", Undergraduate Texts in Mathematics, Springer, 2013.
2. M. Aigner, "A Course in Enumeration", Graduate Texts in Mathematics 238, Springer, 2007.
3. R. P. Stanley, "Enumerative Combinatorics Vol. 2", Cambridge Studies in Advanced Mathematics 62, Cambridge University Press, 1999.

Course Title	**: Foundations of Cryptography**
Course Code	**: MAT474**
Credits	**: 4 Credits**
Course Category	**: Elective**
Course Prerequisites	**: MAT102, MAT206**
Contact Hours	**: 56(including tutorials)**

Outcome of the Course:
Understanding the theoretical study of cryptography which puts foundation for the study and design of real-life cryptography.

Course Contents:
Introduction to cryptography and computational model, computational difficulty, pseudorandom generators, zero-knowledge proofs, encryption schemes, digital signature and message authentication schemes, cryptographic protocol.

References:

1. O. Goldreich, "Foundations of Cryptography - Vol. I and Vol. II", Cambridge University Press, 2001, 2004.
2. S. Goldwasser, Mihir Bellare, "Lecture Notes on Cryptography", 2008, available online from http://cseweb.ucsd.edu/ mihir/papers/gb.html

Course Title	**: Incidence Geometry**
Course Code	**: MAT475**
Credits	**: 4 Credits**
Course Category	**: Elective**
Course Prerequisites	**: MAT205**
Contact Hours	**: 56(including tutorials)**

Outcome of the Course:
Understanding different kinds of incidence structures such as projective spaces, affine spaces, generalized quadrangles, polar spaces and quadratic sets.

Course Contents:
Definitions and Exampleas, projective planes, affine planes, projective spaces, affine spaces, collineations of projective and affine spaces, fundamental theorem of projective and affine spaces, polar spaces, generalized quadrangles, quadrics and quadratic sets.

References:

1. J. Ueberberg, "Foundations of Incidence Geometry", Springer Monographs in Mathematics, Springer, 2011.
2. L. M. Batten, "Combinatorics of Finite Geometries", Cambridge University Press, 1997.
3. E. E. Shult, "Points and Lines", Universitext, Springer, 2011.
4. L. M. Batten, A. Beutelspacher, "The Theory of Finite Linear Spaces: Combinatorics of points and lines", Cambridge University Press, 1993.
5. G. E. Moorhouse, "Incidence Geometry", 2007, available online from http://www.uwyo.edu/moorhouse/handouts/incidence_geometry.pdf

Course Title	**: Lie Algebras**
Course Code	**: MAT476**
Credits	**: 4 Credits**
Course Category	**: Elective**
Course Prerequisites	**: MAT202, MAT205, MAT304**
Contact Hours	**: 56(including tutorials)**

Outcome of the Course:
Understanding the basics of Lie Algebra

Course Contents:
Definitions and Examples, Derivations, Ideals, Homomorphisms, Nilpotent Lie Algebras and Engel's theorem, Solvable Lie Algebras and Lie's theorem, Jordan decomposition and Cartan's criterion, Semisimple Lie algebras, Casimir operator and Weyl's theorem, Representations of $sl(2, F)$, Root space decomposition, Abstract root systems, Weyl group and Weyl chambers, Classification of irreducible root systems, Abstract theory of weights, Isomorphism and conjugacy theorems, Universal enveloping algebras and PBW theorem, Representation theory of semi-simple Lie algebras, Verma modules and Weyl character formula.

References:

1. J. E. Humphreys, "Introduction to Lie Algebras and Representation Theory", Graduate Texts in Mathematics 9, Springer-Verlag, 1978.
2. K. Erdmann, M. J. Wildon, "Introduction to Lie Algebras", Springer Undergraduate Mathematics Series, Springer-Verlag, 2006.
3. J.-P. Serre, "Complex Semisimple Lie Algebras", Springer Monographs in Mathematics, Springer-Verlag, 2001.
4. N. Jacobson, "Lie Algebras", Dover Publications, 1979.

Course Title	**: Optimization Theory**
Course Code	**: MAT477**
Credits	**: 4 Credits**
Course Category	**: Elective**
Course Prerequisites	**: MAT102, MAT205**
Contact Hours	**: 56(including tutorials)**

Outcome of the Course:

Understanding the different techniques used to solve the linear and non-linear programming problem

Course Contents:

Linear programming problem and its formulation, convex sets and their properties, Graphical method, Simplex method, Duality in linear programming, Revised simplex method, Integer programming, Transportation problems, Assignment problems, Games and strategies, Two-person (non) zero-sum games, Introduction to non-linear programming and techniques.

References:

1. J. K. Strayer, "Linear Programming and its Applications", Undergraduate Texts in Mathematics, Springer-Verlag, 1989.
2. P. R. Thie, G. E. Keough, "An Introduction to Linear Programming and Game Theory", John Wiley & Sons, 2008.
3. L. Brickman, "Mathematical Introduction to Linear Programming and Game Theory", Undergraduate Texts in Mathematics, Springer-Verlag, 1989.
4. D. G. Luenberger, Y. Ye, "Linear and Nonlinear Programming", International Series in Operations Research & Management Science 116, Springer, 2008.

Course Title	**: Advanced Partial Differential Equations**
Course Code	**: MAT478**
Credits	**: 4 Credits**
Course Category	**: Elective**
Course Prerequisites	**: MAT401, MAT454**
Contact Hours	**: 56(including tutorials)**

Outcome of the Course:

Learning the basics of distribution Theory, Sobolev Spaces and their properties.

Course Contents:

Distribution Theory, Sobolev Spaces, Embedding theorems, Trace theorem. Dirichlet, Neumann and Oblique derivative problem, Weak formulation, Lax–Milgram, Maximum Principles– Weak and Strong Maximum Principles, Hopf Maximum Principle, Alexandroff-Bakelmann-Pucci Estimate.

References:

1. L. C. Evans, "Partial Differential Equations", Graduate Studies in Mathematics 19, American Mathematical Society, 2010.
2. H. Brezis, "Functional Analysis, Sobolev Spaces and Partial Differential Equations", Universitext, Springer, 2011.
3. R. A. Adams, J. J. F. Fournier, "Sobolev Spces", Pure and Applied Mathematics 140, Elsevier/Academic Press, 2003.
4. S. Kesavan, "Topics in Functional Analysis and Applications", John Wiley & Sons, 1989.
5. M. Renardy, R. C. Rogers, "An Introduction to Partial Differential Equations", Springer, 2008.

Course Title	**: Random Graphs**
Course Code	**: MAT479**
Credits	**: 4 Credits**
Course Category	**: Elective**
Course Prerequisites	**: MAT206, MAT208**
Contact Hours	**: 56(including tutorials)**

Outcome of the Course:
Learning random graphs and their applications.

Course Contents:
Models of random graphs and of random graph processes; illustrative examples; random regular graphs, configuration model; appearance of the giant component small subgraphs; long paths and Hamiltonicity; coloring problems; eigenvalues of random graphs and their algorithmic applications; pseudo-random graphs.

References:

1. N. Alon, J. H. Spencer, "The Probabilistic Method", John Wiley & Sons, 2008
2. B. Bollobás, "Random Graphs", Cambridge Studies in Advanced Mathematics 73, Cambridge University Press, 2001.
3. S. Janson, T. Luczak, A. Rucinski, "Random Graphs", Wiley-Interscience, 2000.
4. R. Durrett, "Random Graph Dynamics", Cambridge University Press, 2010.
5. J. H. Spencer, "The Strange Logic of Random Graphs", Springer-Verlag, 2001.

Course Title	:	**Randomized Algorithms and Probabilistic Methods**
Course Code	:	**MAT480**
Credits	:	**4 Credits**
Course Category	:	**Elective**
Course Prerequisites	:	**MAT206**
Contact Hours	:	**56(including tutorials)**

Outcome of the Course:
Learning how to use probabilistic techniques to different areas of mathematics and computer science.

Course Contents:
Inequalities of Markov and Chebyshev (median algorithm), first and second moment method (balanced allocation), inequalities of Chernoff (permutation routing) and Azuma (chromatic number), rapidly mixing Markov chains (random walk in hypercubes, card shuffling), probabilistic generating functions (random walk in d-dimensional lattice)

References:

1. R. Motwani, P. Raghavan, "Randomized Algorithms", Cambridge University Press, 2004.
2. M. Mitzenmacher, E. Upfal, "Probability and Computing: Randomized algorithms and probabilistic analysis", Cambridge University Press, 2005.

Course Title	**: Statistical Inference I**
Course Code	**: MAT481**
Credits	**: 4 Credits**
Course Category	**: Elective**
Course Prerequisites	**: MAT206, MAT459**
Contact Hours	**: 56(including tutorials)**

Outcome of the Course:

- Understanding about parametric statistical inference to be applicable to almost all branches of statistics.
- Learning various methods of estimation and hypothesis testing and their large sample and small sample properties.

Course Contents:
Review: joint and conditional distributions, order statistics, group family, exponential family. Introduction to parametric inference, sufficiency principle and data reduction, factorization theorem, minimal sufficient statistics, Fisher information, ancillary statistics, complete statistics, Basu's theorem. Unbiasedness, best unbiased and linear unbiased estimator, Rao-Blackwell theorem, Lehmann- Scheffe theorem and UMVUE, Cramer-Rao lower bound and UMVUE, multi-parameter cases. Location and scale invariance, principle of equivarience. Methods of estimation: method of moments, likelihood principle and maximum likelihood estimation, properties of MLE: invariance, consistency, asymptotic normality. Hypothesis testing: error probabilities and power, most powerful tests, Neyman-Pearson lemma and its applications, p-value, uniformly most powerful (UMP) test via Neyman-Pearson lemma, UMP test via monotone likelihood ratio property, existence and nonexistence of UMP test for two sided alternative, unbiased and UMP unbiased tests. Likelihood (generalized) ratio tests and its properties, invariance and most powerful invariant tests. Introduction to confidence interval estimation, methods of fining confidence intervals: pivotal quantity, inversion of a test, examples such as confidence interval for mean, variance, difference in means, optimal interval estimators, uniformly most accurate confidence bound, large sample confidence intervals.

References:

1. E. L. Lehmann and G. Casella, "Theory of Point Estimation", 2nd edition, Springer, New York, 1998.
2. E. L. Lehmann and J. P. Romano, "Testing Statistical Hypothesis", 3rd edition, Springer, 2005.
3. N. Mukhopadhyay, "Probability and Statistical Inference", Marcel Dekker, New York. 2000.
4. G. Casella and R. L. Berger, "Statistical Inference", 2nd edition, Cengage Learning, 2001.
5. A. M. Mood, F. A. Graybill and D. C. Boes, "Introduction to the theory of Statistics", 3rd edition, McGraw Hill, 1974.

Course Title	**: Multivariate Statistical Analysis**
Course Code	**: MAT482**
Credits	**: 4 Credits**
Course Category	**: Elective**
Course Prerequisites	**: MAT205, MAT305, MAT459**
Contact Hours	**: 56(including tutorials)**

Outcome of the Course:

- Learning about various modern statistical tools to analyze and draw inference from multivariate data sets
- Learning inference about multivariate sample mean and variance, techniques of dimension reduction, introductory factor analysis, cluster analysis and statistical pattern recognition.

Course Contents:

Review of matrix algebra (optional), data matrix, summary statistics, graphical representations. Distribution of random vectors, moments and characteristic functions, transformations, some multivariate distributions: multivariate normal, multinomial, Dirichlet distribution, limit theorems. Multivariate normal distribution: properties, geometry, characteristics function, moments, distributions of linear combinations, conditional distribution and multiple correlation. Estimation of mean and variance of multivariate normal, theoretical properties, James-Stein estimator (optional), distribution of sample mean and variance, the Wishart distribution, large sample behavior of sample mean and variance, assessing normality. Inference about mean vector: testing for normal mean, Hotelling T^2 and likelihood ratio test, confidence regions and simultaneous comparisons of component means, paired comparisons and a repeated measures design, comparing mean vectors from two populations, MANOVA. Techniques of dimension reduction, principle component analysis: definition of principle components and their estimation, introductory factor analysis, multidimensional scaling. Classification problem: linear and quadratic discriminant analysis, logistic regression, support vector machine. Cluster analysis: non-hierarchical and hierarchical methods of clustering.

References:

1. K. V. Mardia, J. T. Kent and J. M. Bibby, “Multivariate Analysis”, Academic Press, 1980.
2. T. W. Anderson, “An introduction to Multivariate Statistical Analysis”, Wiley, 2003.
3. C. Chatfield and A. J. Collins, “Introduction to Multivariate Analysis”, Chapman & Hall, 1980.
4. R. A. Johnson and D. W. Wichern, “Applied Multivariate Statistical Analysis”, 6th edition, Pearson, 2007.
5. Brian Everitt and Torsten Hothorn, “An Introduction to Applied Multivariate Analysis with R”, Springer, 2011.
6. M. L. Eaton, “Multivariate Statistics”, John Wiley, 1983.

Course Title	**: Introduction to Manifolds**
Course Code	**: MAT483**
Credits	**: 4 Credits**
Course Category	**: Elective**
Course Prerequisites	**: MAT307**
Contact Hours	**: 56(including tutorials)**

Outcome of the Course:

Knowledge of smooth manifolds, tangent and cotangent spaces, vector bundles, (co)tangent bundles, vector fields, differential forms, exterior differentiation, De-Rham cohomology, integration on manifolds, homotopy invariance of De-Rham cohomology and the statement of Poincare Duality.

Course Contents:

Differentiable manifolds and maps: Definition and examples, Inverse and implicit function theorem, Submanifolds, immersions and submersions. The tangent and cotangent bundle: Vector bundles, (co)tangent bundle as a vector bundle, Vector fields, flows, Lie derivative. Differential forms and Integration: Exterior differential, closed and exact forms, Poincaré lemma, Integration on manifolds, Stokes theorem, De Rham cohomology.

References:

1. Michael Spivak, "A comprehensive introduction to differential geometry", Vol. 1, 3rd edition, 1999.
2. Frank Warner, "Foundations of differentiable manifolds and Lie groups", Springer-Verlag, 2nd edition, 1983.
3. John Lee, "Introduction to smooth manifolds", Springer Verlag, 2nd edition, 2013.
4. Louis Auslander and Robert E. MacKenzie, "Introduction to differentiable manifolds", Dover, 2nd edition, 2009.

Course Title	**: Commutative Algebra**
Course Code	**: MAT484**
Credits	**: 4 Credits**
Course Category	**: Elective**
Course Prerequisites	**: MAT301**
Contact Hours	**: 56(including tutorials)**

Outcome of the Course:
Understanding the various properties of commutative rings, various class of commutative rings, and dimension theory.

Course Contents:
Commutative rings, ideals, operations on ideals, prime and maximal ideals, nilradicals, Jacobson radicals, extension and contraction of ideals, Modules, free modules, projective modules, exact sequences, tensor product of modules, Restriction and extension of scalars, localization and local rings, extended and contracted ideals in rings of fractions, Noetherian modules, Artinian modules, Primary decompositions and associate primes, Integral extensions, Valuation rings, Discrete valuation rings, Dedekind domains, Fractional ideals, Completion, Dimension theory.

Text Book:

1. M. F. Atiyah, I. G. Macdonald, "Introduction to Commutative Algebra", Addison-Wesley Publishing Co., 1969.

References:

1. R. Y. Sharp, "Steps in Commutative Algebra", London Mathematical Society Student Texts, 51. Cambridge University Press, 2000.
2. D. S. Dummit, R. M. Foote, "Abstract Algebra", Wiley-India edition, 2013.

Course Title	**: Algebraic Topology**
Course Code	**: MAT485**
Credits	**: 4 Credits**
Course Category	**: Elective**
Course Prerequisites	**: MAT301, MAT307**
Contact Hours	**: 56(including tutorials)**

Outcome of the Course:

- Understanding the basics of fundamental group (π_1) and singular homology.
- Learning different techniques to compute the fundamental group such as homotopy invariance and Van-Kampen Theorem.
- Learning different techniques to compute singular homology of a space, including homotopy invariance, Mayer-Vietoris, excision, long exact sequence.

Course Contents:
Homotopy Theory: Simply Connected Spaces, Covering Spaces, Universal Covering Spaces, Deck Transformations, Path lifting lemma, Homotopy lifting lemma, Group Actions, Properly discontinuous action, free groups, free product with amalgamation, Seifert-Van Kampen Theorem, Borsuk-Ulam Theorem for sphere, Jordan Separation Theorem. Homology Theory: Simplexes, Simplicial Complexes, Triangulation of spaces, Simplicial Chain Complexes, Simplicial Homology, Singular Chain Complexes, Cycles and Boundary, Singular Homology, Relative Homology, Short Exact Sequences, Long Exact Sequences, Mayer-Vietoris sequence, Excision Theorem, Invariance of Domain.

Text Books:

1. J. R. Munkres, "Topology", Prentice-Hall of India, 2013.
2. A. Hatcher, "Algebraic Topology", Cambridge University Press, 2009.

References:

1. G. E. Bredon, "Topology and Geometry", Graduates Texts in Mathematics 139, Springer, 2009.

Course Title	**: Algebraic Computation**
Course Code	**: MAT551**
Credits	**: 4 Credits**
Course Category	**: Elective**
Course Prerequisites	**: MAT205, MAT304**
Contact Hours	**: 56(including tutorials)**

Outcome of the Course:

It is a unique style of course where the mathematics students having interest in computation can learn to compute different algebraic problems in computer. Here students will learn the computation of the problems related (i) linear algebra, (ii) non-linear system of equations like Grobner bases, (iii) polynomial, (iv) algebraic number theory and (v) elliptic curve.

Course Contents:

Linear algebra and lattices: Asymptotically fast matrix multiplication algorithms, linear algebra algorithms, normal forms over fields, Lattice reduction; Solving system of non-linear equations: Gröbner basis, Buchberger's algorithms, Complexity of Gröbner basis computation; Algorithms on polynomials: GCD, Barlekamp-Massey algorithm, factorization of polynomials over finite field, factorization of polynomials over $\mathbb{Z}$ and $\mathbb{Q}$; Algorithms for algebraic number theory: Representation and operations on algebraic numbers, trace, norm, characteristic polynomial, discriminant, integral bases, polynomial reduction, computing maximal order, algorithms for quadratic fields; Elliptic curves: Implementation of elliptic curve, algorithms for elliptic curves.

References:

1. A. V. Aho, J. E. Hopcroft, J. D. Ullman, "The Design and Analysis of Computer Algorithms", Addison-Wesley Publishing Co., 1975.
2. H. Cohen, "A Course in Computational Algebraic Number Theory", Graduate Texts in Mathematics 138, Springer-Verlag, 1993.
3. D. Cox, J. Little, D. O'shea, "Ideals, Varieties and Algorithms: An introduction to computational algebraic geometry and commutative algebra", Undergraduate Texts in Mathematics, Springer-verlag, 2007.

Course Title	**: Analytic Number Theory**
Course Code	**: MAT552**
Credits	**: 4 Credits**
Course Category	**: Elective**
Course Prerequisites	**: MAT201, MAT207, MAT306**
Contact Hours	**: 56(including tutorials)**

Outcome of the Course:

Learning the elementary properties of Dirichlet series and distribution of primes.

Course Contents:

Arithmetic functions, Averages of arithmetical functions, Distribution of primes, finite abelian groups and characters, Gauss sums, Dirichlet series and Euler products, Reimann Zeta function, Dirichlet L-functions, Analytic proof of the prime number theorem, Dirichlet Theorem on primes in arithmetic progression.

References:

1. T. M. Apostol, "Introduction to Analytic Number Theory", Springer International Student Edition, 2000.
2. K. Chandrasekharan, "Introduction to Analytic Number Theory", Springer-Verlag, 1968.
3. H. Iwaniec, E. Kowalski, "Analytic Number Theory", American Mathematical Society Colloquium Publications 53, American Mathematical Society, 2004.

Course Title	**: Classical Groups**
Course Code	**: MAT553**
Credits	**: 4 Credits**
Course Category	**: Elective**
Course Prerequisites	**: MAT202, MAT205, MAT304**
Contact Hours	**: 56(including tutorials)**

Outcome of the Course:
Understanding the basic facts about classical groups defined over fields such as General Linear groups, Special Linear groups, Symplectic groups, Orthogonal groups and Unitary groups.

Course Contents:
General and special linear groups, bilinear forms, Symplectic groups, symmetric forms, quadratic forms, Orthogonal geometry, orthogonal groups, Clifford algebras, Hermitian forms, Unitary spaces, Unitary groups.

References:

1. L. C. Grove, "Classical Groups and Geometric Algebra", Graduate Studies in Mathematics 39, American Mathematical Society, 2002.
2. E. Artin, "Geometric Algebra", John Wiley & sons, 1988.

Course Title	**: Ergodic Theory**
Course Code	**: MAT554**
Credits	**: 4 Credits**
Course Category	**: Elective**
Course Prerequisites	**: MAT201**
Contact Hours	**: 56(including tutorials)**

Outcome of the Course:
Learning the basics of Ergodic Theory.

Course Contents:
Measure preserving systems; examples: Hamiltonian dynamics and Liouvilles theorem, Bernoulli shifts, Markov shifts, Rotations of the circle, Rotations of the torus, Automorphisms of the Torus, Gauss transformations, Skew-product, Poincare Recurrence lemma: Induced transformation: Kakutani towers: Rokhlins lemma. Recurrence in Topological Dynamics, Birkhoffs Recurrence theorem, Ergodicity, Weak-mixing and strong-mixing and their characterizations, Ergodic Theorems of Birkhoff and Von Neumann. Consequences of the Ergodic theorem. Invariant measures on compact systems, Unique ergodicity and equidistribution. Weyls theorem, The Isomorphism problem; conjugacy, spectral equivalence, Transformations with discrete spectrum, Halmosvon Neumann theorem, Entropy. The Kolmogorov-Sinai theorem. Calculation of Entropy. The Shannon Mc-MillanBreiman Theorem, Flows. Birkhoffs ergodic Theorem and Wieners ergodic theorem for flows. Flows built under a function.

References:

1. Peter Walters, "An introduction to ergodic theory", Graduate Texts in Mathematics, 79. Springer-Verlag, 1982.
2. Patrick Billingsley, "Ergodic theory and information", Robert E. Krieger Publishing Co., 1978.
3. M. G. Nadkarni, "Basic ergodic theory", Texts and Readings in Mathematics, 6. Hindustan Book Agency, 1995.
4. H. Furstenberg, "Recurrence in ergodic theory and combinatorial number theory", Princeton University Press, 1981.
5. K. Petersen, "Ergodic theory", Cambridge Studies in Advanced Mathematics, 2. Cambridge University Press, 1989.

Course Title	**: Harmonic Analysis**
Course Code	**: MAT555**
Credits	**: 4 Credits**
Course Category	**: Elective**
Course Prerequisites	**: MAT302**
Contact Hours	**: 56(including tutorials)**

Outcome of the Course:
knowldege on Fourier Series, Fourier transforms and celebrated differentiation theorem and important operators like Hilbert transform and Maximal function.

Course Contents:
Fourier series and its convergences, Dirichlet kernel, Frejer kernel, Parseval formula and its applications. Fourier transforms,the Schwartz space, Distribution and tempered distribution, Fourier Inversion and Plancherel theorem. Fourier analysis on L_p-spaces. Maximal functions and boundedness of Hilbert transform. Paley-Wiener Theorem for distribution. Poisson summation formula, Heisenberg uncertainty Principle, Wiener's Tauberian theorem.

References:

1. Y. Katznelson, "An Introduction to Harmonic Analysis", Cambridge University Press, 2004.
2. E. M. Stein, G. Weiss, "Introduction to Fourier Analysis on Euclidean Spaces", Princeton Mathematical Series 32, Princeton University Press, 1971.
3. G. B. Folland, "Fourier Analysis and its Applications", Pure and Applied Undergraduate Texts 4, America Mathematical Society, 2010.

Course Title	**: Lie Groups and Lie Algebras - I**
Course Code	**: MAT556**
Credits	**: 4 Credits**
Course Category	**: Elective**
Course Prerequisites	**: MAT205, MAT305, MAT307**
Contact Hours	**: 56(including tutorials)**

Outcome of the Course: Learning the rudiments of Lie groups and irreducible representations of compact Lie groups parametrised by Weyl Character formula.

Course Contents:
General Properties: Definition of Lie groups, subgroups, cosets, group actions on manifolds, homogeneous spaces, classical groups. Exponential and logarithmic maps, Adjoint representation, Lie bracket, Lie algebras, subalgebras, ideals, stabilizers, center Baker-Campbell-Hausdorff formula, Lie's Theorems. Structure Theory of Lie Algebras: Solvable and nilpotent Lie algebras (with Lie/Engel theorems), semisimple and reductive algebras, invariant bilinear forms, Killing form, Cartan criteria, Jordan decomposition. Complex semisimple Lie algebras, Toral subalgebras, Cartan subalgebras, Root decomposition and root systems. Weight decomposition, characters, highest weight representations, Verma modules, Classification of irreducible finite-dimensional representations, BGG resolution, Weyl character formula.

References:

1. D. Bump, "Lie Groups", Graduate Texts in Mathematics 225, Springer, 2013.
2. J. Faraut, "Analysis on Lie Groups", Cambridge Studies in Advanced Mathematics 110, Cambridge University Press, 2008.
3. B. C. Hall, "Lie Groups, Lie algebras and Representations", Graduate Texts in Mathematics 222, Springer-Verlag, 2003.
4. W. Fulton, J. Harris, "Representation Theory: A first course", Springer-Verlag, 1991.
5. J. E. Humphreys, "Introduction to Lie Algebras and Representation Theory", Graduate Texts in Mathematics 9, Springer-Verlag, 1978.
6. A. Kirillov, "Introduction to Lie Groups and Lie Algebras", Cambridge Studies in Advanced Mathematics 113, Cambridge University Press, 2008.
7. V. S. Varadharajan, "Lie Groups, Lie Algebras and their Representations", Springer-Verlag, 1984.

Course Title	**: Operator Algebras**
Course Code	**: MAT557**
Credits	**: 4 Credits**
Course Category	**: Elective**
Course Prerequisites	**: MAT401**
Contact Hours	**: 56(including tutorials)**

Outcome of the Course

Learning the concepts and various structure theorems of C*-algebra and von-Neuman algebra.

Course Contents:

Banach algebras/C*–algebras: Definition and examples; Spectrum of a Banach algebra; Gelfand transform; Gelfand-Naimark theorem for commutative Banach algebras/ C*-algebras; Functional calculus for C*-algebras; Positive cone in a C*-algebra; Existance of an approximate identity in a C*-algebra; Ideals and Quotients of a C*-algebra; Positive linear functionals on a C*-algebra; GNS construction. Locally convex topologies on the algebras of bounded operators on a Hilbert space, von-Neumann's bi-commutant theorem; Kaplansky's density theorem. Ruan's characterization of Operator Spaces (if time permites).

References:

1. R. V. Kadison, J. R. Ringrose, "Fundamentals of the Theory of Operator Algebras Vol. I", Graduate Studies in Mathematics 15, American Mathematical Society, 1997.
2. G. K. Pedersen, "C*–algebras and their Automorphism Groups", London Mathematical Society Monographs 14, Academic Press, 1979.
3. V. S. Sunder, "An Invitation to von Neumann Algebras", Universitext, Springer-Verlag, 1987.
4. M. Takesaki, "Theory of Operator Algebras Vol. I", Springer-Verlag, 2002.

Course Title	**: Representations of Linear Lie Groups**
Course Code	**: MAT558**
Credits	**: 4 Credits**
Course Category	**: Elective**
Course Prerequisites	**: MAT205, MAT305, MAT307**
Contact Hours	**: 56(including tutorials)**

Outcome of the Course:
Learning the first principles of representations and understanding the important examples of 3 different types of groups, viz., compact, nilpotent and solvable groups.

Course Contents:
Introduction to topological group, Haar measure on locally compact group, Representation theory of compact groups, Peter Weyl theorem, Linear Lie groups, Exponential map, Lie algebra, Invariant Differentail operators, Representation of the group and its Lie algebra. Fourier analysis on $SU(2)$ and $SU(3)$. Representation theory of Heisenberg group . Representation of Euclidean motion group.

References:

1. J. E. Humphreys, "Introduction to Lie algebras and representation theory", Springer-Verlag, 1978.
2. S. C. Bagchi, S. Madan, A. Sitaram, U. B. Tiwari, "A first course on representation theory and linear Lie groups", University Press, 2000.
3. Mitsou Sugiura, "Unitary Representations and Harmonic Analysis", John Wiley & Sons, 1975.
4. Sundaram Thangavelu, "Harmonic Analysis on the Heisenberg Group", Birkhauser, 1998.
5. Sundaram Thangavelu, "An Introduction to the Uncertainty Principle", Birkhauser, 2003.

Course Title	**: Harmonic Analysis on Compact Groups**
Course Code	**: MAT559**
Credits	**: 4 Credits**
Course Category	**: Elective**
Course Prerequisites	**: MAT205, MAT401, MAT307**
Contact Hours	**: 56(including tutorials)**

Outcome of the Course:
Knowledge on representaion on compact lie groups with examples $SU(2)$, $SO(n)$.

Course Contents:
Review of General Theory: Locally compact groups, Computation of Haar measure on $\mathbb{R}$, $\mathbb{T}$, $SU(2)$, $SO(3)$ and some simple matrix groups, Convolution, the Banach algebra $L^1(G)$. Representation Theory: General properties of representations of a locally compact group, Complete reducibility, Basic operations on representations, Irreducible representations. Representations of Compact groups: Unitarilzibality of representations, Matrix coefficients, Schur's orthogonality relations, Finite dimensionality of irreducible representations of compact groups. Various forms of Peter-Weyl theorem, Fourier analysis on Compact groups, Character of a representation. Schur's orthogonality relations among characters. Weyl's Chracter formula, Computing the Unitary dual of $SU(2)$, $SO(3)$; Fourier analysis on $SO(n)$.

References:

1. T. Brocker, T. Dieck, "Representations of Compact Lie Groups", Springer-Verlag, 1985.
2. J. L. Clerc, "Les Représentatios des Groupes Compacts, Analyse Harmonique" (J. L. Clerc et. al., ed.), C.I.M.P.A., 1982.
3. G. B. Folland, "A Course in Abstract Harmonic Analysis", CRC Press, 2000.
4. M. Sugiura, "Unitary Representations and Harmonic Analysis", John Wiley & Sons, 1975.
5. E. B. Vinberg, "Linear Representations of Groups", Birkhäuser/Springer, 2010.
6. A. Wawrzyńczyk, "Group Representations and Special Functions", PWN–Polish Scientific Publishers, 1984.

Course Title	**: Modular Forms of One Variable**
Course Code	**: MAT560**
Credits	**: 4 Credits**
Course Category	**: Elective**
Course Prerequisites	**: MAT202, MAT205, MAT207, MAT306**
Contact Hours	**: 56(including tutorials)**

Outcome of the Course:
Learning modular forms over 2 and their congruence subgroups, and their Hecke theory.

Course Contents:
$SL_2(\mathbb{Z})$ and its congruence subgroups, Modular forms for $SL_2(\mathbb{Z})$, Modular forms for congruence subgroups, Modular forms and differential operators, Hecke theory, L-series, Theta functions and transformation formula.

References:

1. J.-P. Serre, "A Course in Arithmetic", Graduate Texts in Mathematics 7, Springer-Verlag, 1973.
2. N. Koblitz, "Introduction to Elliptic Curves and Modular Forms", Graduate Texts in Mathematics 97, Springer-Verlag, 1993.
3. J. H. Bruinier, G. van der Geer, G. Harder, D. Zagier, "The 1-2-3 of Modular Forms", Universitext, Springer-Verlag, 2008.
4. F. Diamond, J. Shurman, "A First Course in Modular Forms", Graduate Texts in Mathematics 228, Springer-Verlag, 2005.
5. S. Lang, "Introduction to Modular Forms", Springer-Verlag, 1995.
6. G. Shimura, "Introduction to the Arithmetic Theory of Automorphic Forms", Princeton University Press, 1994.

Course Title	**: Elliptic Curvese**
Course Code	**: MAT561**
Credits	**: 4 Credits**
Course Category	**: Elective**
Course Prerequisites	**: MAT202, MAT207, MAT306**
Contact Hours	**: 56(including tutorials)**

Outcome of the Course:
Learning elliptic curves and the structure of their rational points.

Course Contents:
Congruent numbers, Elliptic curves, Elliptic curves in Weierstrass form, Addition law, Mordell–Weil Theorem, Points of finite order, Points over finite fields, Hasse-Weil L-function and its functional equation, Complex multiplication.

References:

1. J. H. Silverman, J. Tate, "Rational Points on Elliptic Curves", Undergraduate Texts in Mathematics, Springer-Verlag, 1992.
2. N. Koblitz, "Introduction to Elliptic Curves and Modular Forms", Graduate Texts in Mathematics 97, Springer-Verlag, 1993.
3. J. H. Silverman, "The Arithmetic of Elliptic Curves", Graduate Texts in Mathematics 106, Springer, 2009.
4. A. W. Knapp, "Elliptic Curves", Mathematical Notes 40, Princeton University Press, 1992.
5. J. H. Silverman, "Advanced Topics in the Arithmetic of Elliptic Curves", Graduate Texts in Mathematics 151, Springer-Verlag, 1994.

Course Title	**: Brownian Motion and Stochastic Calculus**
Course Code	**: MAT562**
Credits	**: 4 Credits**
Course Category	**: Elective**
Course Prerequisites	**: MAT472**
Contact Hours	**: 56(including tutorials)**

Outcome of the Course:

Learning about the theory of Brownian motion and it applications to stochastic differential equations.

Course Contents:

Brownian Motion, Martingale, Stochastic integrals, extension of stochastic integrals, stochastic integrals for martingales, Itô's formula, Application of Itô's formula, stochastic differential equations.

References:

1. H. H. Kuo, "Introduction to Stochastic Integration", Springer, 2006.
2. J. M Steele, "Stochastic Calculus and Financial Applications", Springer-Verlag, 2001.
3. F. C. Klebaner, "Introduction to Stochastic Calculus with Applications", Imperial College, 2005.

Course Title	**: Differentiable Manifolds and Lie Groups**
Course Code	**: MAT563**
Credits	**: 4 Credits**
Course Category	**: Elective**
Course Prerequisites	**: MAT305, MAT307**
Contact Hours	**: 56(including tutorials)**

Outcome of the Course:

- Understanding the fundamentals of Lie groups and Lie Algebras.
- Learning about (bi)invariant vector fields, integration on Lie Groups, Cartan's Theorem.

Course Contents:

Review of Several variable Calculus: Directional Derivatives, Inverse Function Theorem, Implicit function Theorem, Level sets in $\mathbb{R}^n$, Taylor's theorem, Smooth function with compact support. Manifolds: Differentiable manifold, Partition of Unity, Tangent vectors, Derivative, Lie groups, Immersions and submersions, Submanifolds. Vector Fields: Left invariant vector fields of Lie groups, Lie algebra of a Lie group, Computing the Lie algebra of various classical Lie groups. Flows: Flows of a vector field, Taylor's formula, Complete vector fields. Exponential Map: Exponential map of a Lie group, One parameter subgroups, Frobenius theorem (without proof). Lie Groups and Lie Algebras: Properties of Exponential function, product formula, Cartan's Theorem, Adjoint representation, Uniqueness of differential structure on Lie groups. Homogeneous Spaces: Various examples and Properties. Coverings: Covering spaces, Simply connected Lie groups, Universal covering group of a connected Lie group. Finite dimensional representations of Lie groups and Lie algebras.

References:

1. D. Bump, "Lie Groups", Graduate Texts in Mathematics 225, Springer, 2013.
2. S. Helgason, "Differential Geometry, Lie Groups and Symmetric Spaces", Graduate Studies in Mathematics 34, American Mathematical Society, 2001.
3. S. Kumaresan, "A Course in Differential Geometry and Lie Groups", Texts and Readings in Mathematics 22, Hindustan Book agency, 2002.
4. F. W. Warner, "Foundations of Differentiable Manifolds and Lie Groups", Graduate Texts in Mathematics 94, Springer-Verlag, 1983.

Course Title	**: Lie Groups and Lie Algebras - II**
Course Code	**: MAT564**
Credits	**: 4 Credits**
Course Category	**: Elective**
Course Prerequisites	**: MAT556**
Contact Hours	**: 56(including tutorials)**

Outcome of the Course:

- Learning the representation theory of compact Lie groups and the group SL(2,C).
- Learning classifications of all simple Lie algebras through root system.

Course Contents:

General theory of representations, operations on representations, irreducible representations, Schur's lemma, Unitary representations and complete reducibility. Compact Lie groups, Haar measure on compact Lie groups, Schur's Theorem, characters, Peter-Weyl theorem, universal enveloping algebra, Poincare-Birkoff-Witt theorem, Representations of Lie($SL(2, \mathbb{C})$). Abstract root systems, Weyl group, rank 2 root systems, Positive roots, simple roots, weight lattice, root lattice, Weyl chambers, simple reflections, Dynkin diagrams, classification of root systems, Classification of semisimple Lie algebras. Representations of Semisimple Lie algebras, weight decomposition, characters, highest weight representations, Verma modules, Classification of irreducible finite-dimensional representations, Weyl Character formula, The representation theory of $SU(3)$, Frobenius Reciprocity theorem, Spherical Harmonics.

References:

1. D. Bump, "Lie Groups", Graduate Texts in Mathematics 225, Springer, 2013.
2. J. Faraut, "Analysis on Lie Groups", Cambridge Studies in Advanced Mathematics 110, Cambridge University Press, 2008.
3. B. C. Hall, "Lie Groups, Lie algebras and Representations", Graduate Texts in Mathematics 222, Springer-Verlag, 2003.
4. W. Fulton, J. Harris, "Representation Theory: A first course", Springer-Verlag, 1991.
5. A. Kirillov, "Introduction to Lie Groups and Lie Algebras", Cambridge Studies in Advanced Mathematics 113, Cambridge University Press, 2008.
6. A. W. Knapp, "Lie Groups: Beyond an introduction", Birkäuser, 2002.
7. B. Simon, "Representations of Finite and Compact Groups", Graduate Studies in Mathematics 10, American Mathematical Society, 2009.

Course Title	**: Mathematical Foundations for Finance**
Course Code	**: MAT565**
Credits	**: 4 Credits**
Course Category	**: Elective**
Course Prerequisites	**: MAT472**
Contact Hours	**: 56(including tutorials)**

Outcome of the Course:
Learning about the mathematical modeling of simple stock markets and techniques to analyze them.

Course Contents:
Financial market models in finite discrete time, Absence of arbitrage and martingale measures, Valuation and hedging in complete markets, Basic facts about Brownian motion, Stochastic integration, Stochastic calculus: Itô's formula, Girsanov transformation, Itô's representation theorem, Black-Scholes formula

References:

1. J. Jacod, P. Protter, "Probability Essentials", Universitext, Springer-Verlag, 2003.
2. D. Lamberton, B. Lapeyre, "Introduction to Stochastic Calculus Applied to Finance", Chapman-Hall, 2008.
3. H. Föllmer, A. Schied, "Stochastic Finance: An Introduction in Discrete Time", de Gruyter, 2011.

Course Title	**: Designs and Codes**
Course Code	**: MAT566**
Credits	**: 4 Credits**
Course Category	**: Elective**
Course Prerequisites	**: MAT205, MAT304**
Contact Hours	**: 56(including tutorials)**

Outcome of the Course:
Understanding the technique used for constructing combinatorial designs and its relation with linear codes.

Course Contents:
Incidence structures, affine planes, translation plane, projective planes, conics and ovals, blocking sets. Introduction to Balanced Incomplete Block Designs (BIBD), Symmetric BIBDs, Difference sets, Hadamard matrices and designs, Resolvable BIBDs, Latin squares. Basic concepts of Linear Codes, Hamming codes, Golay codes, Reed-Muller codes, Bounds on the size of codes, Cyclic codes, BCH codes, Reed-Solomon codes.

References:

1. G. Eric Moorhouse, "Incidence Geometry", 2007 (available online).
2. Douglas R. Stinson, "Combinatorial Designs", Springer-Verlag, New York, 2004.
3. W. Cary Huffman, V. Pless, "Fundamentals of Error-correcting Codes", Cambridge Uinversity Press, Cambridge, 2003.

Course Title	**: Statistical Inference II**
Course Code	**: MAT567**
Credits	**: 4 Credits**
Course Category	**: Elective**
Course Prerequisites	**: MAT481**
Contact Hours	**: 56(including tutorials)**

Outcome of the Course:

- Learning decision theory and Bayesian estimation and testing.
- Learning about large sample theory including asymptotic tests, confidence intervals, asymptotic efficiency and optimality of estimators and tests.

Course Contents:
General decision problem, loss and risk function, minimax estimation, minimaxity and admissibility in exponential family. Introduction to Bayesian estimation, Bayes rule as average risk optimality, prior and posterior, conjugate families, generalized Bayes rules. Bayesian intervals and construction of credible sets, Bayesian hypothesis testing. Empirical and nonparametric empirical Bayes analysis, admissibility of Bayes and generalized Bayes rules, discussion on Bayes versus non-Bayes approaches. Large sample theory: review of modes of convergences, Slutsky's theorem, Berry-Essen bound, delta method, CLT for iid and non iid cases, multivariate extensions. Asymptotic level α tests, asymptotic equivalence, comparison of tests: relative efficiency, asymptotic comparison of estimators, efficient estimators and tests, local asymptotic optimality. Bootstrap sampling: estimation and testing.

References:

1. E. L. Lehmann and G. Casella, "Theory of Point Estimation", 2nd edition, Springer, New York, 1998.
2. E. L. Lehmann, "Elements of Large-Sample Theory", Springer-Verlag, 1999.
3. E. L. Lehmann and J. P. Romano, "Testing Statistical Hypothesis", 3rd edition, Springer, 2005.
4. James O Berger, "Statistical Decision Theory and Bayesian Analysis", 2nd edition, Springer, New York, 1985.

Course Title	**: Ordered Linear Spaces**
Course Code	**: MAT568**
Credits	**: 4 Credits**
Course Category	**: Elective**
Course Prerequisites	**: MAT401**
Contact Hours	**: 56(including tutorials)**

Outcome of the Course:
Understanding the vector order structure and its relation with Functional Analysis.

Course Contents:
Cones and orderings; order convexity; order units; approximate order units; bases. Positive linear mappings and functionals; extension and separation theorems; decomposition of linear functionals into positive linear functionals. Vector lattices; basic theory. Norms and orderings; duality of ordered spaces; (approximate) order unit spaces; base normed spaces. Normed and Banach lattices; AM-spaces, AL-spaces; Kakutani theorems for AM-spaces and AL-spaces. Matrix ordered spaces: matricially normed spaces; matricial ordered normed spaces; matrix order unit spaces; Arveson-Hahn-Banach extension theorem.

Recommended books:

1. G.J.O. Jameson, "Lecture Notes in Mathematics" 141 Springer-Verlag, 1970.
2. N.C. Wong and K.F. Ng, "(2) Partially ordered topological vector spaces", Oxford University Press, 1973.
3. C.D. Aliprantis and O. Burkinshaw, "Positive operators", Academic Press, 1985.
4. H.H. Schaefer, "Banach lattices and positive operators", Berlin: Springer, 1974.

References:

1. W.A. J. Luxemburg and A.C. Zaanen, "Riesz Spaces", Elsevier, 1971.
2. A.C. Zaanen, "Introduction to operator theory in Riesz spces (Vol 1 & Vol 2)", Springer, 1997

Course Title : **Topics in H^p Spaces**
Course Code : **MAT569**
Credits : **4 Credits**
Course Category : **Elective**
Course Prerequisites : **MAT302, MAT306**
Contact Hours : **56(including tutorials)**

Outcome of the Course:

- Understanding analytic and harmonic functions on the unit disc.
- Understanding properties H^p spaces, for $1 \leq p < \infty$.
- Understanding invarint subspaces for the shift operator on H^2 space.

Course Contents:
Fourier Series: Cesaro Means, Characterization of Types of Fourier Series; Analytic and Harmonic Functions in the Unit Disc: The Cauchy and Poisson Kernels, Boundary Values, Fatou's Theorem, H^p Spaces; The Space H^1: The Helson-Lowdenslager Approach, Szego's Theorem, Completion of the Discussion of H^1; Factorization for H^p functions: Inner and Outer Functions, Blaschke Products and Singular Functions, The Factorization Theorem, Absolute Convergence of Taylor Series, Functions of Bounded Characteristic; Analytic Functions with Continuous Boundary Values: Conjugate Harmonic Functions, Theorems of Fatou and Rudin; The Shift Operator: The Shift Operator on H^2, Invariant Subspaces for H^2 of the Half-plane, Isometries, The Shift Operator on L^2.

References:

1. Kenneth Hoffman, Banach spaces of analytic functions, Reprint of the 1962 original, Dover Publications, Inc., New York, 1988.
2. Walter Rudin, Real and complex analysis, Third edition. McGraw-Hill Book Co., New York, 1987.
3. Duren, Peter L., Theory of H^p spaces, Pure and Applied Mathematics, Vol. 38 Academic Press, New York-London 1970.

Course Title	**: Introduction to Dilation Theory**
Course Code	**: MAT570**
Credits	**: 4 Credits**
Course Category	**: Elective**
Course Prerequisites	**: MAT401**
Contact Hours	**: 56(including tutorials)**

Outcome of the Course:

- Understanding contractive operators by exhibiting as a compression of unitary operators.
- Understanding Hardy classes and H^∞ space.
- Understanding dilations of commuting and non-commuting contractions.

Course Contents:
Contractions and Their Dilations: Unilateral shifts, Wold decomposition, Bilateral shifts, Contractions, Canonical decomposition, Isometric and unitary dilations, Matrix construction of the unitary dilation, a discussion on rational dilation; Geometrical and Spectral Properties of Dilations: Structure of the minimal unitary dilations, Isometric dilations, Dilation of commutants; Functional Calculus: Hardy classes, Inner and outer functions, The classes H^∞ and H^∞_T, The role of outer functions, Contractions of class C_0; Operator-Valued Analytic Functions: The spaces $L^2(\mathcal{U})$ and $H^2(\mathcal{U})$, Inner and outer functions, Lemmas on Fourier representation, Factorizations, Analytic kernels; Functional Models: Characteristic functions, Functional models for a given contraction, Functional models for analytic functions; A discussion on Commuting and non-commuting contractions and their dilations.

References:

1. Béla Sz.-Nagy, Ciprian Foias, Hari Bercovici and László Kérchy, Harmonic analysis of operators on Hilbert space; Second edition. Revised and enlarged edition. Universitext. Springer, New York, 2010.

2. Vern Ival Paulsen, Completely bounded maps and operator algebras, Cambridge Studies in Advanced Mathematics, 78. Cambridge University Press, Cambridge, 2002.

3. Jim Agler, John Harland and Benjamin J. Raphael, Classical function theory, operator dilation theory, and machine computation on multiply-connected domains. (English summary) Mem. Amer. Math. Soc. 191 (2008), no. 892.

4. Nikolai K. Nikolskii, Operators, functions, and systems: an easy reading. Vol. 1 and Vol 2. Hardy, Hankel, and Toeplitz, American Mathematical Society, Providence, RI, 2002.

Course Title	**: Arithmetic of Quadratic Forms**
Course Code	**: MAT571**
Credits	**: 4 Credits**
Course Category	**: Elective**
Course Prerequisites	**: MAT201, MAT205, MAT207, MAT304**
Contact Hours	**: 56(including tutorials)**

Outcome of the Course: This first part of the course shall give an overview of classical theory of Binary quadratic forms, computation of class numbers, understanding of composition rule and correspondence between binary quadratic forms and classes of ideals in quadratic fields. The second part will give a more general theory of quadratic forms over various fields and local-global principle.

Course Contents:

I. Binary Quadratic forms-Elementary concepts, Modular groups and its fundamental domain, Positive definite forms and reduction, Indefinite forms and reduction, Automorphs and Pells equation; The class group: Representation and Genera, Composition of forms, Class number computations; Quadratic number fields and Ideals, Binary quadratic forms and Classes of ideals.

II. Quadratic forms over any field K, with $char(K) \neq 2$, Isotropic elements and hyperbolic planes, Witts extension Theorem; p-adic fields and Hilbert symbol; Quadratic forms over $\mathbb{Q}_p$, Hasse-Minkowski Theory of Quadratic forms over $\mathbb{Q}$, Quadratic forms with prescribed invariants.

    Some applications: Legendres Theorem (on representation of zero by $aX^2+bY^2+cZ^2$), Gausss Theorem (Sum of three squares), Lagranges Theorem (Sum of Four Squares).

References:

1. Binary Quadratic forms by D. A. Buell (Springer)(Chapter 17).
2. A Course on Arithmetic, J.-P. Serre (Springer) (Part-I).
3. Algebra: Volume 3 by Luther, Passi (Narosa) (Chapter 5).
4. The Arithmetic Theory of Quadratic Forms by Buton. W. Jones (The Carus Mathematical Monographs).
5. Rational Quadratic Forms by J.W.S. Cassels (Dover Publications).
6. Number Theory, Vol. I by Henri Cohen (Springer)
7. An Introduction to the Theory of Numbers by I. Niven, H. S. Zuckerman and H. L. Montgomery (Wiley)
8. Number Theory by Z. I Borevich and I. Shafarevich (Academic Press)
9. Elementary Number Theory by E. Landau (Chelsea Publishing Company)

Course Title	**: Optimization and optimal control**
Course Code	**: MAT572**
Credits	**: 4 Credits**
Course Category	**: Elective**
Course Prerequisites	**: MAT303, MAT204, MAT205**
Contact Hours	**: 56(including tutorials)**

Outcome of the Course: The main aim of this course is to familiarize the students with fundamentals of variational calculus in function spaces and optimal control of ordinary differential equations. By the end of this course, students will have an in-depth knowledge of these topics and will be ready for more advanced courses in calculus of variations.

Course Contents:
Calculus in normed linear spaces, convexity and fundamental theorems of optimization, Euler Lagrange equations, examples (the brachistochrone problem, minimal surface area of revolution).
Controllability: linear case and nonlinear autonomous systems, Bang-Bang principle, Existence theorems for Optimal Control problems.
Necessary conditions for Optimal Controls: The Pontryagin Maximum Principle.

References:

1. J. Macki and A. Strauss, Introduction to Optimal Control Theory, New York: Springer; 1982.

2. J.L. Troutman, Variational Calculus with Elementary Convexity, New York: Springer; 1983.

3. A. Sasane, Optimization in function spaces, Dover Publications, 2016.

4. D.G. Luenberger, Optimization by Vector Space methods. Wiley, 1969.

5. I.M. Gelfand and S.V. Fomin, Calculus of Variations. Dover, 1963.

6. J. Zabczyk, Mathematical control theoryan introduction. Second edition, Systems Control Found. Appl., Birkhuser/Springer, Cham, 2020.

7. H.O. Fattorini, Infinite-dimensional optimization and control theory. Encyclopedia of Mathematics and its Applications, 62. Cambridge University Press, Cambridge, 1999.

8. W. Liu, Elementary Feedback Stabilization of the Linear Reaction-Convection-Diffusion Equation and the Wave Equation, Mathmatiques & Applications (Berlin) [Mathematics & Applications], 66. Springer-Verlag, Berlin, 2010.

9. A. Bensoussan, G. Da Prato, M.C. Delfour, S.K. Mitter, Representation and control of infinite dimensional systems. Second edition. Systems & Control: Foundations & Applications. Birkhauser Boston, Inc., Boston, MA, 2007.

10. L.C. Evans, Mathematical Methods for Optimization: Finite Dimensional Optimization, Lecture notes.

11. L.C. Evans, Mathematical Methods for Optimization: Dynamic Optimization, Lecture notes.

12. L.C. Evans, An Introduction to Mathematical Optimal Control Theory, Lecture notes.

Course Title	**: Introduction to Coxeter groups**
Course Code	**: MAT573**
Credits	**: 4 Credits**
Course Category	**: Elective**
Course Prerequisites	**: MAT202, MAT205**
Contact Hours	**: 56(including tutorials)**

Outcome of the Course: Coxeter groups find applications in many areas of mathematics. Studying these groups involves the interplay of geometry, algebra, and combinatorics. This course aims to familiarize students with the general theory of Coxeter groups (mostly from a combinatorial approach). Upon successful completion of the course, students will be familiar with various examples of finite and affine Coxeter groups and finite reflection groups. They will also learn the linearity of Coxeter groups, Exchange properties, Root system, Hecke algebra, Classification of finite and affine Coxeter groups, and Automorphisms of Coxeter groups.

Course Contents:
Coxeter systems and Coxeter groups, Coxeter graphs, matrices and corresponding Artin groups, Irreducible Coxeter systems, Types of Coxeter groups and Artin groups: Spherical, 2-Spherical, Crystallographic, Right-angled, Universal, Large, Simply laced, Even, Odd, etc., Various examples of finite and affine Coxeter groups, and Artin groups, Parabolic subgroups, Length function.
Linearity of Coxeter groups, Root system, Permutation representation, Exchange properties, and Deletion conditions. Bruhat order.
Finite reflection groups, Polynomial Invariants of finite reflection groups, Hecke Algebras, Kazhdan- Lusztig, and R-polynomials.
Classification of finite and Affine Coxeter groups, Automorphisms of Coxeter groups.
Additional Topics: (If time permits, then some of the following topics may be covered.) Isomorphism problem in Coxeter groups, $K(\Pi, 1)$ conjecture for Artin groups, Enumeration in Coxeter groups, Stanley symmetric function.

References:

1. A. Bjorner, and F. Brenti, Combinatorics of Coxeter Groups, Springer, 2005.
2. J. E. Humphreys, Reflection groups and Coxeter groups, Cambridge University Press, 1990.
3. Richard Kane, Reflection Groups and Invariant theory, CMS Books in Mathematics (Springer-Verlag), 2001.
4. P. Bahls, The isomorphism problem in Coxeter Groups, Imperial College Press, 2005.
5. Nicolas Bourbaki, Elements Of Mathematics: Lie Groups and Lie Algebras: Chapters 4-6, Springer 2002.
6. Michael W. Davis, The Geometry and Topology of Coxeter Groups, Princeton University Press, 2008.

Course Title	**: Introduction to Homological Algebra**
Course Code	**: MAT574**
Credits	**: 4 Credits**
Course Category	**: Elective**
Course Prerequisites	**: MAT202, MAT301**
Contact Hours	**: 56(including tutorials)**

Outcome of the Course: The first part of this course shall give an overview of Category theory and functors. Different types of covariant and contravariant functors, such as Hom, tensor, Tor, and Ext, will be described. In the second part, the homology and cohomology of groups will be introduced, and spectral sequence will be discussed.
This course aims to familiarize the students with categories, functors, group cohomology, spectral sequences and commutative diagrams. By the end of these courses, students will have a working knowledge of this topic.

Course Contents:

I. Categories, Functors, Abelian categories, Category of modules, Hom and Tensor functors. Localization, Projective and Injective modules, Derived functors, Tor and Ext functors.

II. Group rings, G-modules, Bar resolution, Homology and cohomology of groups, low dimensional homologies, Universal coefficient theorem, Second cohomology group and Extensions, Kunneth Formula, The Schur Multiplier and its properties, Transgression, Restriction and Inflation homomorphisms, Hochschild-Serre spectral sequence and applications.

References:

1. Joseph J. Rotman, An introduction to Homological algebra, New York: Springer; 2009.

2. Weibel, Charles A. An introduction to homological algebra. No. 38. Cambridge University Press, 1995.

3. Cartan, Henry, and Samuel Eilenberg. Homological algebra. Vol. 41. Princeton university Press, 1999.

4. Maclane, Saunders. Categories for the working mathematician. Vol. 5. Springer Science & Business Media, 2013.

5. MacLane, Saunders. Homology. Springer Science & Business Media, 2012.

6. Hilton, Peter J., and Urs Stammbach. A course in homological algebra. Vol. 4. Springer Science & Business Media, 2012.

7. L.R. Vermani, An elementary approach to homological algebra. CRC Press, 2003.

8. Grothendieck, A. (1957), Sur quelques points d'algbre homologique, Thoku Mathematical Journal, (2), 9: 119–221. (Grothendieck Tohoku paper)