EM induction experiment to determine the moment of a magnet

2014 Phys. Educ. 49 319

(http://iopscience.iop.org/0031-9120/49/3/319)

EM induction experiment to determine the moment of a magnet

K M Najiya Maryam

Department of Physics, PSMO College, Tirurangadi, Kerala, India

E-mail: najiya.maryam@gmail.com

Abstract

If we drop a magnet through a coil, an emf is induced in the coil according to Faraday's law of electromagnetic induction. Here, such an experiment is done using expEYES kit. The plot of emf versus time has a specific shape with two peaks. A theoretical analysis of this graph is discussed here for both short and long cylindrical magnets. Mathematical expressions are derived for both. Knowing this equation, experiments to calculate the moment of a magnet can be devised. If we use a long conducting tube instead of a simple coil in this experiment, it can even help in measuring the eddy current damping coefficient k.

Introduction

According to Faraday's law of EM induction, whenever the flux linked with a coil changes, an emf is induced in the coil which is proportional to the rate of change of flux linkage. Also, Lenz's law states that the emf is induced in such a direction as to oppose the change in flux. So this emf causes an eddy current to be developed in the coil.

The flux through a coil can be changed in many ways, such as moving the coil in a magnetic field or moving a magnet back and forth through the coil. Here we discuss the case in which the coil is fixed and the magnet is dropped through it from a height. The emf induced in the coil versus time can be examined with a cathode-ray oscilloscope or any other measuring device. The curve is obtained on a computer screen using the expEYES kit, giving two peaks, as shown in figure 1. Here we have used a cylindrical magnet having 10 mm length and diameter 5 mm and a coil having 3000 turns. The values of the peak voltages are −2.560 and 2.832 V and are separated by 13 ms. The amplitude of the graph will be different if we use a magnet having a different strength or if we drop the magnet from a different height.

We can also note that the second peak is bigger than the first peak, due to the acceleration of the magnet as it moves down, showing that the emf depends on the velocity of the magnet. We can also see that the emf increases with an increase in the moment of the magnet. If we can obtain a mathematical expression for the induced emf as a function of time then the above experiment can be used to measure the moment of the magnet.

Mathematical expression

The derivation of the expression will be easier if consider the magnet to be a current-carrying coil or a solenoid.

0031-9120/14/030319+07$33.00

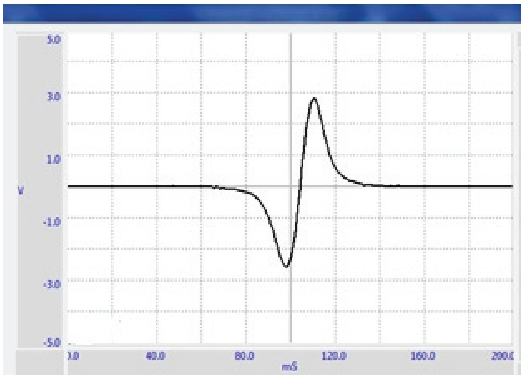

Figure 1. Induced emf versus time curve obtained experimentally using expEYES kit.

If the magnet is small we can approximate it as a plane current-carrying coil; otherwise we can approximate it as a current-carrying solenoid.

For a small magnet

The induced emf is given by

$$\text{emf} = -N\frac{\mathrm{d}\Phi}{\mathrm{d}t} \tag{1}$$

according to Faraday's law, where Φ is the flux.

$$\text{emf} = -N\frac{\mathrm{d}(BA)}{\mathrm{d}t} \qquad [\because B \text{ is } \Phi/A] \tag{2}$$

where A is the area of the coil, N is the number of turns of the coil and B is the magnetic field produced due to the small cylindrical bar magnet at the centre of the coil.

Firstly, we will consider the case in which the magnet is moved through the coil with a constant velocity using some mechanism.

We can consider the magnet with a dipole moment m as a current-carrying loop having n turns if the length of the magnet is appreciably small.

$$\begin{aligned} m &= nIA = nI\pi R^2 \\ nIR^2 &= \frac{m}{\pi} \end{aligned} \tag{3}$$

where R is the radius of the cylindrical magnet.

We know that the field along the axis of the circular coil is given by [1]

$$B = \frac{\mu_o n I R^2}{2}(R^2 + x^2)^{-\frac{3}{2}},$$

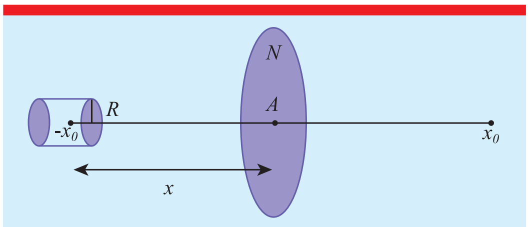

Figure 2. A schematic representation of a small magnet moving horizontally through a coil.

where x is the distance from the centre of the magnet to the centre of the coil

$$B = \frac{\mu_o m}{2\pi}(R^2 + x^2)^{-\frac{3}{2}}$$

which in terms of m from (3) leads to

$$\begin{aligned} \text{emf} &= -NA\frac{\mathrm{d}B}{\mathrm{d}t} \\ &= -\frac{\mu_o m}{2\pi}NA\frac{\mathrm{d}}{\mathrm{d}t}(R^2 + x^2)^{-\frac{3}{2}} \\ &= -\frac{\mu_o m}{2\pi}NA\frac{-3}{2}(R^2 + x^2)^{-\frac{5}{2}}2x\frac{\mathrm{d}x}{\mathrm{d}t} \\ &= \frac{3\mu_o m}{2\pi}NA(R^2 + x^2)^{-\frac{5}{2}}xv \\ &\quad \times \left[\because \frac{\mathrm{d}x}{\mathrm{d}t} = v, \text{ velocity of the magnet}\right]. \end{aligned} \tag{4}$$

Here we consider the coil to be located at the origin and the magnet to be moving horizontally along the x-direction, as shown in figure 2. So x varies from some $-x_o$ to x_o. The corresponding graph of emf versus position is shown in figure 3(a).

Since v is constant,

$$x = -x_o + vt$$

can be substituted into (4) to give

$$\begin{aligned} \text{emf} = \frac{3\mu_o m}{2\pi}&NA(R^2 + (-x_o + vt)^2)^{-\frac{5}{2}} \\ &\times\ v(-x_o + vt), \end{aligned} \tag{5}$$

which is plotted in figure 3(b).

If the velocity is constant we can see that both the peaks will have the same value.

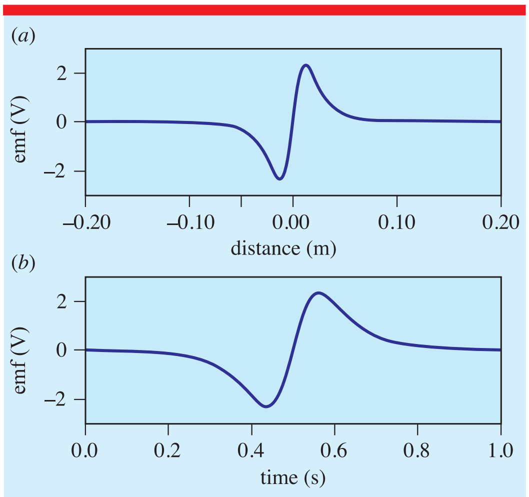

Figure 3. (a) Emf versus position plotted according to equation (4), (b) emf versus time plotted according to equation (5).

However, in general, the velocity will not be a constant due to air resistance and gravitational forces unless there is some external mechanism.

If we drop the magnet vertically along the z-direction keeping the coil at origin, as shown in figure 4,

$$z = -z_o + \tfrac{1}{2}gt^2$$
$$v = gt$$

where $-z_o$ is the initial position of the magnet.

Here g is the acceleration of the magnet. On passing through the coil, g decreases due to eddy current damping. Eddy current damping is responsible for the time delay in magnets falling through a long conductor. If the coil is short we can neglect this. So the emf will be

$$\text{emf} = \frac{3\mu_o m}{2\pi} NA\left(-z_o + \frac{1}{2}gt^2\right) \times (R^2 + (-z_o + \tfrac{1}{2}gt^2)^2)^{-\frac{5}{2}} gt. \quad (6)$$

The corresponding graph is plotted for given parameter values in figure 5.

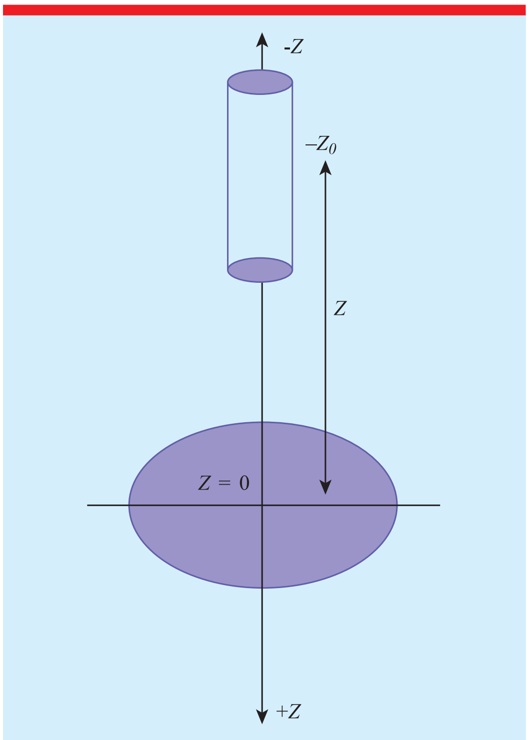

Figure 4. A schematic representation of a magnet falling vertically through a coil.

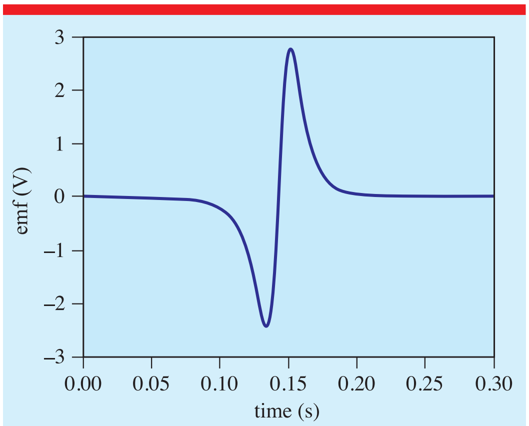

Figure 5. Emf versus time graph for a vertically dropping magnet according to equation (6).

For a long magnet

If the magnet is long enough it cannot be considered as a simple coil, but should rather be considered as a solenoid with n turns. Since the magnet we are considering is a cylindrical magnet, it can be easily approximated as a solenoid.

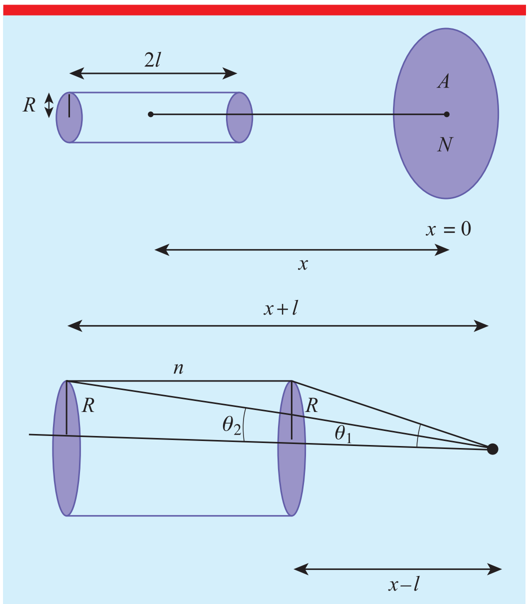

Figure 6. A schematic representation of a long magnet moving horizontally through a coil.

The field along the axis of a solenoid is given by [1]

$$B = \frac{\mu_o n_l I}{2}(\cos\theta_2 - \cos\theta_1)$$
$$= \frac{\mu_o I n}{2\times 2l}((x+l)(R^2+(x+l)^2)^{-\frac{1}{2}} - (x-l)(R^2+(x-l)^2)^{-\frac{1}{2}})$$

where x is the distance from the centre of the coil to the centre of the magnet, as shown in figure 6. l is the half length of the magnet, R is the radius of the cylindrical magnet and 'n_l' is the number of turns per unit length ($n_l = \frac{n}{2l}$).

$$\Phi = BA = A\frac{\mu_o n I}{2\times 2l}((x+l)(R^2+(x+l)^2)^{-\frac{1}{2}} - (x-l)(R^2+(x-l)^2)^{-\frac{1}{2}}).$$

But $nI\pi R^2 = m$, the moment of the magnet.

Writing this in terms of m, the moment of the magnet, from (3), applying Faraday's law (1) and differentiating—as described in greater detail in the appendix—we get:

$$\text{emf} = -N\frac{\mathrm{d}\Phi}{\mathrm{d}t} = -NAm\frac{\mu_o}{4\pi l}v[((x+l)^2+R^2)^{-\frac{3}{2}} - ((x-l)^2+R^2)^{-\frac{3}{2}}]. \quad (7)$$

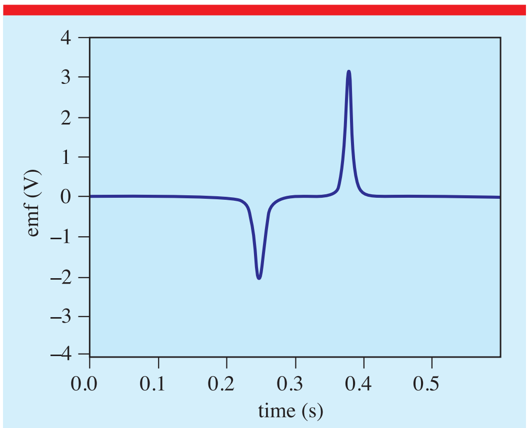

Figure 7. Emf versus time plotted according to equation (8) for a long magnet.

This equation can be used if the velocity is constant.

However, if the magnet is falling vertically, let us take the direction of motion of the magnet to be in the positive z-direction, as shown in figure 4. Let the coil be at the origin $z = 0$ and the initial position of the magnet (the point at which we are dropping the magnet) be $-z_o$ at time $t = 0$.

So at any time t

$$z = -z_o + \tfrac{1}{2}gt^2$$
$$v = \frac{\mathrm{d}z}{\mathrm{d}t} = gt.$$

So the emf induced is

$$\text{emf} = -\frac{NAm\mu_o}{4\pi l}gt[(R^2+(-z_o+\tfrac{1}{2}gt^2+l)^2)^{-\frac{3}{2}} - (R^2+(-z_o+\tfrac{1}{2}gt^2-l)^2)^{-\frac{3}{2}}]. \quad (8)$$

The corresponding graph is shown in figure 7.

This is indeed the general equation, which can be used even for small magnets. If l is small, the graph corresponding to (8) is shown in figure 8.

The emf versus time graph can be easily obtained on a computer screen using the expEYES/phoenix kit, as shown in figure 1 [2]. The mathematical expression for this is given in (8). So, by curve fitting, we can easily determine the value of the magnetic moment m, proving that the em induction experiment can be used to determine the moment of a magnet.

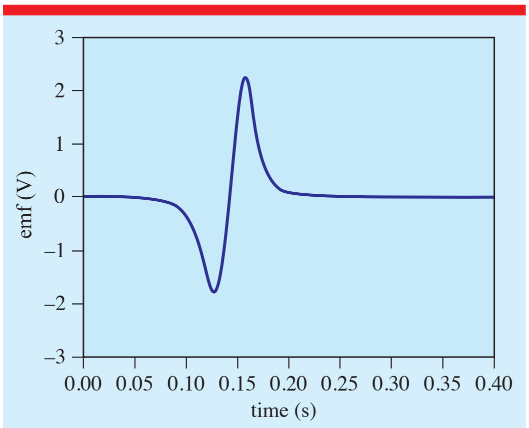

Figure 8. Emf versus time plotted according to equation (8) for a small magnet.

Effect of the eddy current damping—a theoretical study

Since we are using a small coil here, the acceleration of the magnet will not deviate much from g. It will be less than but close to g. However, if we use a long hollow conductor instead, then while passing through the conductor the acceleration of the magnet decreases and finally becomes zero, i.e. the magnet reaches its constant terminal velocity. This is due to the eddy current damping. The equation of motion of the magnet (as long as the magnet is inside the conductor) is given by

$$ma = mg - kv$$

where m is the mass of the magnet, a is the acceleration of the magnet inside the conductor, g is the acceleration due to gravity, k is the electromagnetic damping coefficient and v is the instantaneous velocity of the magnet.

The above equation of motion $m\frac{dv}{dt} = mg - kv$ is a differential equation with the initial condition $v(0) = v_1$ (velocity of magnet while entering the conductor). The solution of this differential equation is

$$v = \frac{mg}{k} + \left(v_1 - \frac{mg}{k}\right)\exp\left(-\frac{k}{m}t\right)$$

and

$$a = g - \frac{k}{m}v = \left(g - \frac{k}{m}v_1\right)\exp\left(-\frac{k}{m}t\right)$$

[by substituting the value of v].

If $v_1 = 0$ (i.e. we drop the magnet just above the conductor), then

$$v = \frac{mg}{k}\left(1 - \exp\left(-\frac{k}{m}t\right)\right)$$

$$a = g\exp\left(-\frac{k}{m}t\right).$$

So here the acceleration is a function of time, as long as the magnet is inside the conductor. It will certainly be less than g. If the conductor is long enough, a changes from 9.8 to 0 m s^{-2}. But for small coils or conductors having short lengths, this change is negligible, as the magnet spends a very short time inside the conductor. So in our previous case (i.e. the experiment with the cylindrical magnet and a coil with small length) we assumed that a is approximately equal to g.

If we use a long hollow conductor, this change in acceleration of magnet with time (while the magnet is inside the conductor) should also be taken into account. A python program is written to accommodate this effect and graphs of emf versus time for different values of k (eddy current damping coefficient) are plotted, as shown in figure 9. It can be seen that as k changes the general shape of the curve does not change, but the peaks occur at different points and the value of the peak voltage also changes, i.e. the position and magnitude of the peak voltage depend on the value of k. So, with the help of the em induction experiment using a long hollow conductor instead of the coil, the specific curve can be obtained. This may serve as an experiment to determine the electromagnetic damping coefficient by the method of curve fitting. This may even help in studying the parameters of the magnet and the conductor on which the value of k depends.

Acknowledgments

The author acknowledges Mr B P Ajith Kumar, IUAC New Delhi for giving her inspiration to do this work. The author also acknowledges her cousin Ms K Aysha Hadya for helping her to type this paper.

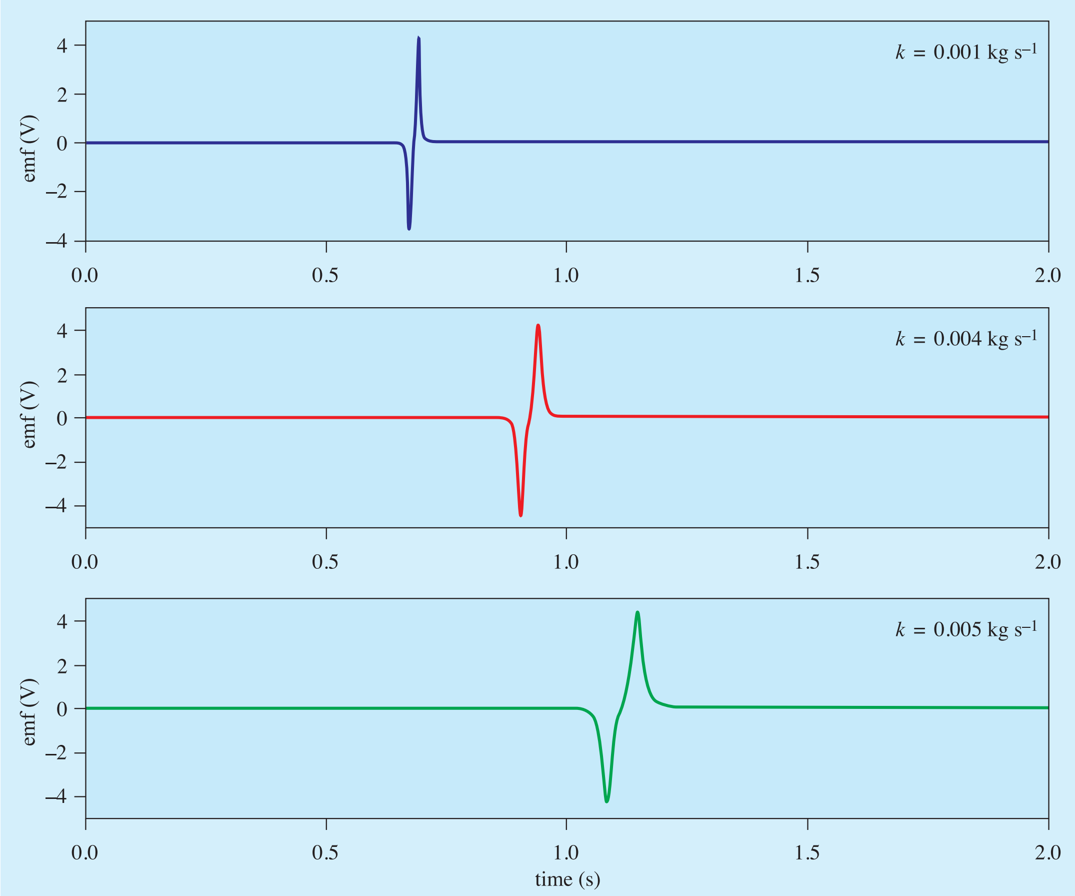

Figure 9. Emf versus time graphs for three different values of k plotted using a Python program.

Appendix

$$\Phi = \frac{A\mu_o}{2\times 2}\frac{m}{\pi R^2 l}[(x+l)(R^2+(x+l)^2)^{-\frac{1}{2}} - (x-l)(R^2+(x-l)^2)^{-\frac{1}{2}}]$$

$$\text{emf} = -N\frac{\mathrm{d}\Phi}{\mathrm{d}t}$$

$$= -\frac{NAm}{lR^2}\frac{\mu_o}{4\pi}\Bigg[((x+l)^2+R^2)^{-\frac{1}{2}}\frac{\mathrm{d}x}{\mathrm{d}t} - \tfrac{1}{2}(x+l)((x+l)^2+R^2)^{-\frac{3}{2}} \times 2(x+l)\frac{\mathrm{d}x}{\mathrm{d}t} - ((x-l)^2+R^2)^{-\frac{1}{2}} \times \frac{\mathrm{d}x}{\mathrm{d}t} - \frac{1}{2}2(x-l)((x-l)^2+R^2)^{-\frac{3}{2}} \times 2(x-l)\frac{\mathrm{d}x}{\mathrm{d}t}\Bigg]$$

$$= -\frac{NAm}{lR^2}\frac{\mu_o}{4\pi}v[((x+l)^2+R^2)^{-\frac{1}{2}} - ((x-l)^2+R^2)^{-\frac{1}{2}} - (x+l)^2 \times ((x+l)^2+R^2)^{-\frac{3}{2}} + (x-l)^2((x-l)^2+R^2)^{-\frac{3}{2}}]$$

$$= -\frac{NAm}{lR^2}\frac{\mu_o}{4\pi}v[((x+l)^2+R^2) \times ((x+l)^2+R^2)^{-\frac{3}{2}} - (x+l)^2((x+l)^2+R^2)^{-\frac{3}{2}} - ((x-l)^2+R^2)((x-l)^2+R^2)^{-\frac{3}{2}}$$

$$
\begin{aligned}
&- (x-l)^2((x-l)^2+R^2)^{-\frac{3}{2}}] \\
&= -\frac{NAm}{lR^2}\frac{\mu_o}{4\pi}v[((x+l)^2+R^2)^{-\frac{3}{2}} \\
&\times ((x+l)^2+R^2-(x+l)^2) \\
&- ((x-l)^2+R^2)^{-\frac{3}{2}}((x-l)^2 \\
&+ R^2-(x-l)^2)] \\
&= -\frac{NAm}{lR^2}\frac{\mu_o}{4\pi}v[R^2((x+l)^2+R^2)^{-\frac{3}{2}} \\
&- R^2((x-l)^2+R^2)^{-\frac{3}{2}}] \\
&= -NAm\frac{\mu_o}{4\pi l}v[((x+l)^2+R^2)^{-\frac{3}{2}} \\
&- ((x-l)^2+R^2)^{-\frac{3}{2}}].
\end{aligned}
$$

Received 22 July 2013, in final form 19 August 2013
doi:10.1088/0031-9120/49/3/319

References

[1] Griffiths D J 1999 *Introduction to Electrodynamics* 3rd edn (New Delhi: Pearson)
[2] *expEYES Junior Users manual; IUAC New Delhi* (http://expeyes.in/sites/default/files/Documents/eyesj-a4.pdf)

Najiya Maryam K M is working as an assistant professor at PSMO College, Tirurangadi, Kerala. She has completed her degree from Farook College, Calicut and PG from Indian Institute of Technology, Madras.