Study of Magnetic Resonance with a Compass

Magnetic resonance is a quantum phenomenon and describes a resonant interaction of spins with electromagnetic fields. Magnetic resonance plays an essential role in the fields of medicine, science and technology. Magnetic resonance imaging is an indispensable noninvasive tool in medical diagnostics and research. Nuclear magnetic resonance spectroscopy is regularly used in physics, chemistry, biology, and materials sciences for investigating physical and chemical properties and fingerprinting of substances.

Goal: Understanding the concept of magnetic resonance through a simple table-top experiment.

Description: We place a compass on the table. The needle obviously will align with the Earth's magnetic field. If we bring a permanent magnet (much stronger than Earth's magnetic field) close to the compass, the needle will realign towards the magnet. Same effect will take place if we place the compass near an electromagnetic coil (constant current). However, if we supply a weak alternating current through the coil, the needle will not follow the alternating magnetic field and instead oscillate along the initial alignment of the permanent magnet (If both permanent magnet and em coil is present). The oscillation remains very small when the permanent magnet is very close or far away. At an intermediate distance between the compass and the permanent magnet, the oscillation becomes very strong. This is the indication of the resonance.

Theory:

Consider the magnetic moment of the needle to be $\bar{m}$. In the combined magnetic field of the permanent magnet ($\bar{B}_{\mathrm{PM}}$) and the Earth ($\bar{B}_{\oplus}$), the needle experiences a torque

$$\mathcal{T} = \bar{m} \times (\bar{B}_{\mathrm{PM}} + \bar{B}_{\oplus})\,. \tag{0.1}$$

As noted earlier, the permanent magnet is much stronger than the Earth's magnetic field and hence the latter can be neglected. Let's denote by $\bar{B}_{\mathrm{drive}}$ the alternating drive field produced by the coil with alternating current. The alternating drive field will make the needle oscillate in the compasplane. The compass lies in the horizontal plane in the center of the coil. The permanent magnet is aligned along the x-axis.

The sum of the torques due to the field of the permanent magnet and due to the coil, is equal to the time-derivative of the angu- lar momentum of the needle:

$$\bar{m} \times (\bar{B}_{\mathrm{PM}} + \bar{B}_{\mathrm{drive}}) = \ddot{\theta} J \hat{z} \tag{0.2}$$

where J is the moment of inertia of the compass needle. θ is the deflection angle. The z-component of the previous equation reads

$$mB_{\mathrm{PM}} \sin(-\theta) + mB_{\mathrm{drive}} \sin\left(\frac{\pi}{2} - \theta\right) = \ddot{\theta} J\,. \tag{0.3}$$

We can consider small angle approximation i.e. $\sin(-\theta) \equiv -\theta$ and $\sin\left(\frac{\pi}{2} - \theta\right) \equiv 1$. We find

$$\ddot{\theta} + \omega_0^2 \theta = \omega^2\,, \tag{0.4}$$

where

$$\omega_0 = \sqrt{\frac{mB_{\mathrm{PM}}}{J}} \quad \text{and} \quad \omega = \sqrt{\frac{mB_{\mathrm{drive}}}{J}}\,. \tag{0.5}$$

Eqn. 0.4 is a differential equation for a driven harmonic oscillator with the resonance frequency $\frac{\omega_0}{2\pi}$. When both the frequencies (ω and ω_0) matches, the resonance condition happens and the needle experiences the resonant excitation i.e. the oscillation amplitude drastically increases.

In the experiment, we keep the frequency of the coil fixed. We slide the permanent magnet closer to or farther from the compass and cahnge the field $\bar{B}_{\text{PM}}$ experienced by the compass. The field of the permanent magnet in the dipole approximation depends on the distance d between the centers of the magnet and the compass as

$$B_{\text{PM}} = \frac{\mu_0}{4\pi} \frac{2m_{\text{PM}}}{d^3} \,. \tag{0.6}$$

m_{PM} is the magnetic moment of the permanent magnet. Finally the resonance frequency is

$$f_{res} = \frac{1}{2\pi} \sqrt{\frac{\mu_0}{2\pi} \frac{m_{\text{PM}}}{d^3} \frac{m}{J}} \,. \tag{0.7}$$

Procedure:

This experiment contains a circular coil, a permanent magnet, a function generator and a compass. The procedure is as follows:

1 Arrange the permanent magnet such that magnetic fields of coil and permanent magnet are perpendicular.

2 Apply a sine wave of frequency in the range of 1 to 3 Hz with 20 dB button pressed in the level button of given function generator.

3 For a given distance of the magnet from the coil, slowly adjust the frequency and determine the resonance frequency.

4 Magnetic field versus distance data will be provided to you.

5 Plot the magnetic field versus resonant frequency.

References:
1.https://pubs.aip.org/aapt/pte/article/57/9/633/612253/Exploring-Magnetic-Resonance-with-a-Compass
2.https://news.ucr.edu/articles/2019/12/05/simple-experiment-explains-magnetic-resonance