

ELECTROMAGNETIC DAMPING OF A COMPOUND PENDULUM

Objective: To study the electromagnetic damping of a compound pendulum.

Introduction:

Damping plays an important role in controlling the motion of an object. It is an effect, which tends to reduce the velocity of a moving object. A number of damping techniques are used in various moving, oscillating and rotating systems. These techniques include conventional friction damping, air friction damping, fluid friction damping and electromagnetic (eddy current) damping. Electromagnetic damping is one of the most interesting damping techniques, which uses electromagnetically induced currents to slow down the motion of a moving object without any physical contact with the moving object.

To understand the phenomenon of electromagnetic damping, we need to know about electromagnetic induction (discovered by Michael Faraday in 1831) and eddy currents (also known as Foucault currents - discovered by Leon Foucault in 1851). Electromagnetic induction is a phenomenon in which an electromotive force (emf) is induced in a conductor when it experiences a changing magnetic field. An emf is induced when either the conductor moves across a steady magnetic field or when the conductor is placed in a changing magnetic field. Due to this induced emf and the conducting path available, induced currents (flow of electrons) are set up in the body of the conductor. These induced currents are in the form of 'eddy currents' which are electrons swirling within the body of the conductor like water swirling in a whirlpool (eddy).

The eddy currents swirl in such a way as to create a magnetic field opposing the change in the magnetic field experienced by the conductor in accordance with Lenz's law. Thus the eddy currents swirl in a plane perpendicular to the magnetic field. These eddy currents interact with the magnetic field to produce a force, which opposes the motion of the moving conductor or object. The damping force increases as the distance of the conductor from the magnet decreases. This damping force is also proportional to the strength of the magnetic field and the induced eddy currents and hence the velocity of the object. Thus faster the object moves the stronger is the damping force. This means that as the object slows down, the damping force is reduced, resulting in a smooth stopping motion.

In this experiment, a magnet is attached to a compound pendulum and a metal sheet(plate) is placed at a certain distance from the magnet. The metal sheet should be placed in such a way that it is parallel to the plane of oscillation and perpendicular to the length of the magnet as shown in Figure 1. While the pendulum oscillates, the magnetic flux passing through the metal will change and induce eddy current in the metal plate.

Apparatus

Compound pendulum with a pointer, tripod stand, set of magnets, copper plate with holder, mirror, graph paper to mark position of pointer, stopwatch.

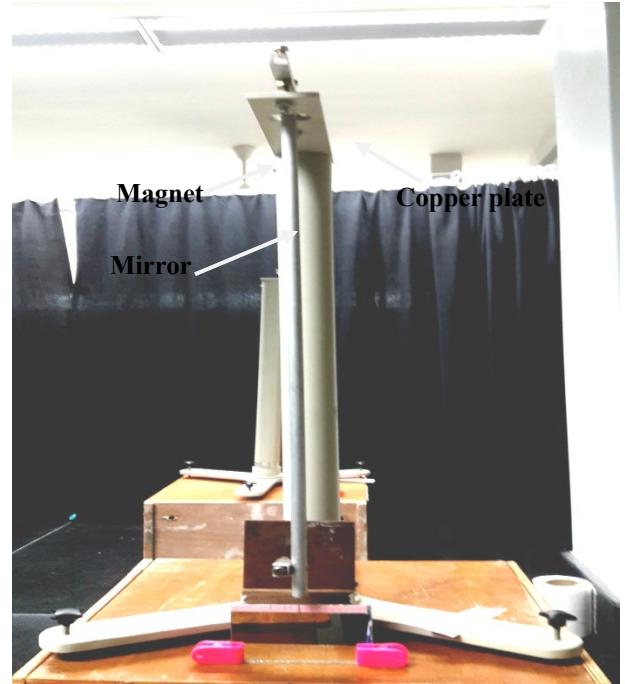
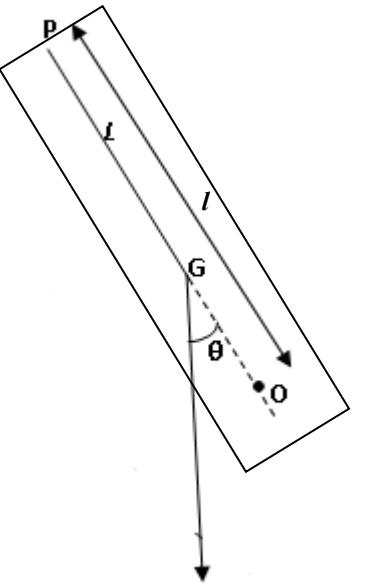


Fig. 1: Experimental set up

Theory

Consider a compound pendulum pivoted about a horizontal frictionless axis through P and displaced from its equilibrium position by an angle θ (see Fig. 2). In the equilibrium position the center of gravity G of the body is vertically below P. The distance GP is L, the mass of the body is m and I is the moment of inertia of the body through the axis P. Then the equation of motion for small amplitude oscillation is given by

$$I\ddot{\theta} = -mgL\theta \quad \dots \dots (1)$$



Thus the solution of Eq. 1 becomes,

$$\theta = \theta_0 \sin(\omega_0 t) \dots \dots \dots (2)$$

Fig. 2: Compound Pendulum

where θ_0 is the maximum angular amplitude, $\omega_0 = 2\pi/\sqrt{\frac{mgL}{I}}$. So time period for free oscillation of the pendulum is given by

$$T_0 = 2\pi\sqrt{\frac{I}{mgL}} \quad \dots \dots \dots (3)$$

When the oscillation is damped due to any resistance in the path such as friction or eddy current etc, the damping force exerted at the free end of the rod is directly proportional to velocity, v , of the free end. Let γ be the constant of proportionality called the damping coefficient. Then this force can be written as

$$F = -\gamma v = -\gamma l\omega \quad \dots \quad \dots \quad (4)$$

where l = actual length of the rod between P and O, ω = angular velocity and the negative sign indicates that the force is always directed opposite to the velocity. Then the torque is given by

$$lF = -\gamma l^2\omega = -\gamma l^2\dot{\theta} \quad \dots \quad \dots \quad (5)$$

Thus the equation of motion for a damped oscillation is given by

$$I\ddot{\theta} = -mgL\theta - \gamma l^2\dot{\theta} \quad \dots \quad \dots \quad (6)$$

The solution of this modified equation is

$$\theta = \theta_0 e^{-\frac{t}{\tau}} \cos(\omega_1 t) \quad \dots \quad \dots \quad (7)$$

where $\tau = \frac{2I}{\gamma l^2}$, is called the decay constant and ω_1 is the angular frequency of damped oscillation.

Thus, θ becomes maximum (but $< \theta_0$ due to the exponential decay function in Eq. 7), for $\omega_1 t = 2n\pi$, where $n = 0, 1, 2, \dots$. If T_1 is the time period of this damped oscillator, then it attains maximum displacement at times, $t = nT_1$. Hence the maximum displacement of the pendulum decreases exponentially with time as given by the following equation:

$$\theta_{max} = \theta_0 e^{-\frac{t}{\tau}} \quad \dots \quad \dots \quad (8)$$

Equation 8 can be rewritten in terms of an equation of a straight line as follows:

$$\ln\left(\frac{\theta_{max}}{\theta_0}\right) \approx \ln\frac{x_n}{x_0} = -\left(\frac{T_1}{\tau}\right)n \quad \dots \quad \dots \quad (9)$$

where x_0 is the initial and x_n is the final linear amplitude after n oscillations. Knowing τ from the slope of the straight line, the damping coefficient can be calculated as

$$\gamma = \frac{2I}{\tau l^2} \quad \dots \quad \dots \quad (10)$$

Procedure

1. The compound pendulum with permanent magnets is mounted on a tripod stand with a two pin pivot arrangement. Make sure that the pendulum does not slip from the pivot.

2. Place a mirror vertically very close to the pin attached to the pendulum as a pointer. Adjust the position so that the image of the tip of the pin can be seen in the mirror avoiding parallax error. A graph sheet is pasted on the mirror to mark and note the linear amplitudes of the pendulum. Mark the equilibrium position on the graph sheet.
3. Determine the time period of the pendulum without any damping. While pendulum oscillates without any plate, observe the amplitude damping due to air resistance and friction.
4. Place the copper plate at a distance, say about 15 mm, so that the plane of the copper plate is perpendicular to the axis of the magnets (see Fig.1).
5. Measure the time period for 10 oscillations by using a stop watch provided. Repeat it for 3 to 5 times to find the average time period of oscillation.
6. Displace the pendulum from the mean position to a position of initial amplitude, x_0 (say about 20mm), and then leave it to oscillate. Note the final amplitude after 2 oscillations. Repeat it for at least three times, and then find the average value of x_2 .
7. Repeat the above step for 4, 6, 8, 10 oscillations keeping the initial position fixed.
8. Fill up the observation table and plot a graph between the number of oscillations, $\ln \frac{x_n}{x_0}$ vs n . Determine the slope using straight line fit and find decay time τ .
9. Replace the copper plate with aluminum plate and maintain same distance between magnet and plate, repeat steps 5 to 8.
10. Finally, calculate the damping co-efficient ' γ ' using the given values of I and l for both the metals.

Observations

Given: Moment of inertia of the supplied rod, $I = 0.0235 \text{ kgm}^2$

Length of the rod, $l = 0.61\text{m}$

Table 1: Time period of oscillation of pendulum without any damping

Sl. No.	Time for 10 Oscillations (sec)	Time period T (sec)	Average time period T (sec)
1			
2			
3			
4			
5			

Table 2: Time period of damped oscillation with copper plate $d = \dots\dots$

Sl. No.	Time for 10 Oscillations (sec)	Time period T_1 (sec)	Average time period T_1 (sec)
1			
2			
3			
4			
5			

Table 3: Measurement of amplitude with damping with copper plate $x_0 = \dots\dots \text{ mm}$

No. of Oscillations (n)	Sl. No.	x_n (mm)	x_n/x_0	$\ln(x_n/x_0)$
2	1			
	2			
	3			
4	4			
	5			
	6			
6	7			
	8			
	9			
8	10			
	11			
	12			
10	13			
	14			
	15			

Table 4: Time period of damped oscillation with aluminum plate

Sl. No.	Time for 10 Oscillations (sec)	Time period T_2 (sec)	Average time period T_2 (sec)
1			
2			
3			
4			
5			

Table 5: Measurement of amplitude with damping with aluminum plate

No. of Oscillations (n)	Sl. No.	x_n (mm)	x_n/x_0	$\ln(x_n/x_0)$
2	1			
	2			
	3			
4	4			
	5			
	6			
6	7			
	8			
	9			
8	10			
	11			
	12			
10	13			
	14			
	15			

Graph: Plot $\ln \frac{x_n}{x_0}$ vs n and straight line fit to determine slope and intercept for both experiments with copper and aluminum and error in slope and intercept.

Calculations: $\tau = \dots$

$$\Delta\tau = \dots$$

$$\gamma = \dots$$

$$\Delta\gamma = \dots$$

Precautions:

1. Avoid parallax error while noting the amplitude.
2. Mark the amplitude carefully on the graph sheet pasted on the mirror without disturbing the set up.