# Measurement of Elementary charge by Millikan oil drop method

## Aims:

The aims of this experiment are to demonstrate that electrical charge is quantised in discrete multiples of the electronic charge, e, and to measure the value of e.

## Introduction:

This experiment aims at measuring the charge of an electron and is perhaps the most basic of all atomic physics or modern physics laboratory experiments. It won Millikan the Nobel Prize in the year 1923.

The experiment depends on the ability to control, measure and balance very small force of the order $10^{-14}$ N. The set-up consists of two horizontal parallel plates separated by about 5 mm. The upper plate has a small hole through which microscopic oil droplets are sprayed in between the two plates with the help of an atomizer which is like a common perfume sprayer. These droplets get charged due to the frictional force during spraying. The free fall of these droplets in the space between the plates is observed in the gravitational field. A measurement of the velocity of fall along with the use of Stokes law leads to the calculation of the mass of the droplets and their radii if the oil density is known. These are of the order of $10^{-15}$ kg and $10^{-6}$ m respectively. By applying a potential difference between the plates, a uniform electric field is produced in the space between the plates. A measurement of the velocity of the negatively charged droplets rising in the electric field allows a calculation of the electric force on the droplets and hence the charge carried by them. In the experiment the droplets which rise and fall slowly are selected as they are expected to have a fairly small charge. These droplets are made to rise and fall several times. The repetitions of measurement of the velocities of rise and fall reduce the random error of their means. A fairly large number of droplets are observed and their charges are calculated.

The analysis of the data on the total charge carried by the droplets shows that these total charges are integral multiples of a certain smallest charge which is the charge of an electron. This result also shows that the charge is quantized.

The measurement of the charge on the electron can lead to the calculation of Avogadro's number. The charge F (the Faraday) required to electro-deposit one gram equivalent of an element on an electrode during electrolysis is equal to the charge of the electron multiplied by the number of molecules in a mole. The

Faraday has been found to be $F = 9.625 \times 10^{7}$ coulombs per kilogram equivalent weight. Hence Avogadro's number $N = F/e$. Apparatus Required:

The present set-up consists of

1. A oil drop chamber mounted on top of the panel. It has

(i) A pair of horizontal parallel plate electrodes separated by about 5 mm thick ebonite ring with a hole for viewing the oil droplets.

(ii)The upper plate has a small hole in its centre for the admission of the droplets which are produces by spraying oil with an atomizer.

(iii)A device to illuminate the space between the plate electrodes.

2.Three levelling screws at the base of the panel to make the parallel plate electrodes perfectly horizontal (perpendicular to the gravitational field) and a water-level placed on top of the panel to check it.

3.A microscope with CCD camera head to view and transmit image of oil droplets between the plate electrodes to the monitor.

4.A power pack to supply continuously variable voltage in the range 0 – 800 V to the upper plate electrode when the electric field is to be created between the plates. The lower plate is permanently grounded.

5.A digital voltmeter to measure the potential applied to the upper plate.

6. A 'Time Meter' to display the time for which the oil droplet is allowed to move.

7.A timing device to measure time interval between the passage of droplets through preset points. There are two keys to operate this device. Pressing the 'Clear' key, wipes out the time information. This is like a reset key. The time meter now reads '00.0' sec. Pressing the 'Start/Stop' key starts the timing device. Pressing it again stops the device. The elapsed time can be read on the meter.

8.A monitor with graduated screen. The horizontal lines on the monitor screen help in setting the distance through which the droplets move.

9.An atomizer to spray droplets.

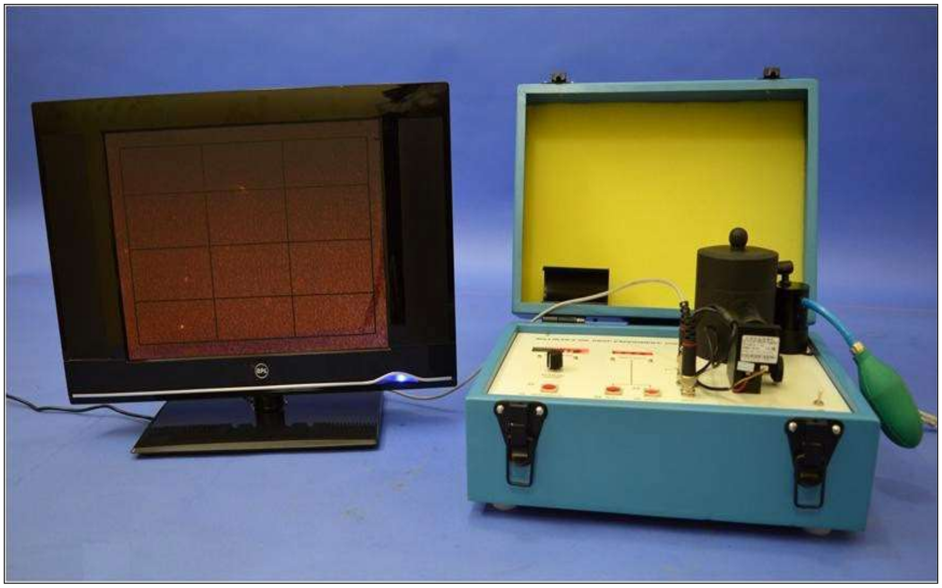

**Figure 1: Millikan oil drop experiment set up**

**Technical Specifications:**

1. Gap between parallel plates: 5 mm
2. Voltage supply between the plates: 140 – 800 V DC continuously variable
   - Regulation: ± 0.1 %
   - Stability: ± 1 digit

3. Voltmeter: 3 ½ digit LED
4. Illumination between the plated: LED
5. Microscope: Magnification 30 X
6. Time Meter /; 0-999.9 sec
7. Monitor: 420 TVL, LCD monitor
8. Input Power Supply: 220 V AC/50 Hz
9. Density of the oil used: 929 $Kg/m^3$

**Theory:**

Consider a spherical oil droplet of radius ‘r’ and density ‘ρ’ falling under the gravitational force. This droplet in air is acted upon by a constant force and soon reaches a terminal velocity given by Stoke’s law, $F_\eta = 6\pi\eta r v_f$, where ‘η’ is the coefficient of viscosity of air and ‘$v_f$’ is the terminal velocity during the fall. The gravitational and buoyancy forces acting on the droplet are balanced by $F_\eta$, Figure 2 (a).

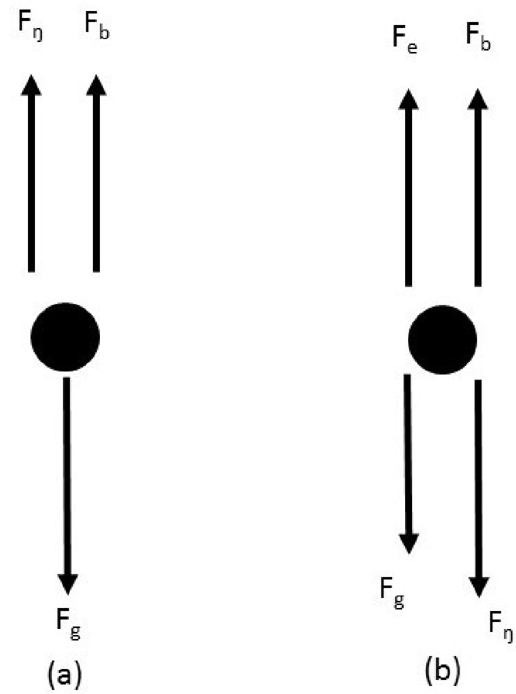


Figure 2. Different forces acting on the oil droplet (a) Forces $F_g$, $F_b$ and $F_\eta$ on the droplet due to gravity, buoyancy and viscosity, respectively during the free fall. (b) Forces $F_g$, $F_b$, $F_\eta$ and $F_e$ on the droplet due to gravity, buoyancy, viscosity and the electric field during the rise of the droplet.

$$\frac{4}{3}\pi r^3 \rho g - \frac{4}{3}\pi r^3 \rho_a g = 6\pi\eta r v_f \quad \ldots\ldots.(1)$$

Here, $\rho_a$ is the density of air. The falling velocity $v_f$ is thus given by

$$v_f = \frac{2}{9}\frac{gr^2}{\eta}(\rho - \rho_a) \ldots\ldots\ldots\ldots\ldots..(2)$$

If the droplet carries a charge ne and is moving upward with terminal velocity $v_f$ under the influence of the applied electric field V/d between the parallel plate electrodes separated by the distance d and potential difference, the force equation is, Figure 2 (right)

$$\frac{4}{3}\pi r^3 (\rho - \rho_a) g + 6\pi\eta r v_r = \frac{Vne}{d} \ldots\ldots\ldots\ldots.(3)$$

Subtracting Eq.(1) from Eq.(3) and solving for ne, we get

$$ne = \frac{6\pi\eta r d}{V}(v_f - v_r) \ldots\ldots\ldots..(4)$$

Dividing Eq.(4) by Eq.(2), gives

$$ne = \frac{4\pi}{3}\frac{gd}{V}\,(\rho - \rho_a)\, r^3\left(1 + \frac{v_r}{v_f}\right) \ldots\ldots\ldots(5)$$

The Stoke's law used in obtaining Eqs.(1) –(5) assumes that the droplets are moving slowly, that there is no slipping of the medium over the surface of the droplet, that the medium is of quite large extent compared to the size of the droplet and that the inhomogeneities in the medium are of a size small compared to the size of the droplets. In the present case all the assumptions except the last one are reasonably valid. The radii of the droplets are of the order of one micron and therefore not much greater than the mean free path of the air molecules. The droplets will tend to fall more quickly in the free space between the air molecules. The expression for the falling velocity $v_f$ corrected for this effect on the basis of kinetic theory is

$$v_f = \frac{2}{9}\frac{gr^2}{\eta}(\rho - \rho_a)\left(1+\frac{C}{Pr}\right) \ldots\ldots\ldots\ldots\ldots\ldots(6)$$

Where, C (=6.17 x $10^{-8}$ m of Hg-m) is a correction factor and P (in m of Hg) is the atmospheric pressure. Writing

$$\xi = \frac{9\eta}{2g} v_f / (\rho - \rho_a) \ldots\ldots\ldots\ldots\ldots\ldots(7)$$

$$\zeta = \frac{C}{2P} \ldots\ldots\ldots\ldots\ldots(8)$$

Eq.(6) now reduces to

$$r^2 + 2\zeta r - \xi = 0 \ldots\ldots\ldots\ldots\ldots(9)$$

The radius of the droplet is given by the positive root of this equation

$$r = -\zeta + \sqrt{\zeta^2 + \xi} \ldots\ldots\ldots(10)$$

The charge ne may be obtained by first calculating $\xi$ and $\zeta$ from Eqs. (7) and (8), then calculating the radius 'r 'from Eq. (10) and finally ne from Eq.(5).

Equation (5) above is for the Dynamic method.

In the Balancing method, the droplet is kept stationary by adjusting the potential. The upward velocity is thus equal to zero. The final equation for ne in the Balancing method is therefore

$$ne = \frac{4\pi}{3}\frac{gd}{V_b}\,(\rho - \rho_a)\, r^3 \;......(11)$$

where, $V_b$ is the balancing potential

Time for free-fall of the droplets under gravity between the preset points is measured to obtain the velocity for free- fall. For observing the effect of the electric field on the charge carried by the droplets, there are two alternatives.

In the Dynamic method, the velocity for the vertical upward motion is measured for a fixed potential difference.

The result of this unit are within 5% of the standard value.

**Procedure:**

1. Note the atmospheric pressure and the room temperature.
2. Switch on the power supply
3. Wait for about ten minutes to let the system stabilize.
4. Fix two horizontal lines on the monitor. These are the preset lines. The rise and fall times of the droplets move from one line to the other are to be recorded. A good choice is to choose the second line from the top and the last line from the bottom. This will leave some room at the top and at the bottom for manoeuvring and preventing the droplet from getting lost.
5. Press the 'clear key' to make the 'time meter' read zero.
6. Spray droplets of oil from the atomizer. One or two are squirts are usually sufficient.
7. As the droplet drift down, some of them pass through the hole in the upper parallel plate and reach the region in-between the plates where they are illuminated.
8. These are viewed by the microscope/camera as unsolved points of diffracted light and the images are transmitted to the monitor screen.
9. These droplets drift down slowly under gravitation force.
10. Under the influence of the electric field between the plates, the motion of the droplets will get affected if they are charged. If a droplet moves downward more slowly under the influence of the electric force (corresponding to the upper plate at a positive potential) the drop is negatively charged. Its motion can be arrested or even reversed (made to rise) by an increase in the potential. If the downward drifting of the droplet increases under the action of electric force, the droplet is positively charged. Such droplets are ignored in the present set-up.

11. Suitable negatively charged droplet is selected. ‘Suitable’ means that it drifts downward slowly (about 10-15 s free-fall time) under gravitational field indicating that the droplet is not too heavy. Its mass should also not be too small, otherwise it will bounce around due to random collisions with air molecules. Brownian motion and it will difficult to estimate when the droplet actually crosses a line. A very small droplet may cross a line 10 - 20 times. It should also be possible to make it rise by applying a voltage of about 500 V indicating that there are only a few charge quanta on the droplet. Fix some value of the voltage.
12. This section is done by concentrating on one or two droplets and removing all others from the field by switching off the electric field.
13. Once a droplet has been chosen, measurement can begin. There are two approaches

a. **Dynamic:**

Measure the free-fall time with the voltage off and the rise time with some suitable voltage on for the droplet to move between the two above chosen lines on the monitor. For these measurements following procedure may be followed.

(i) Pull the droplet above the top chosen line on the monitor by adjusting the voltage.
(ii) Press the ‘clear key’ to make the ‘time meter ‘read zero.
(iii) Switch off the voltage, the droplet will begin to fall freely
(iv) Start the ‘time meter’ by pressing the ‘start/stop’ key when the droplet crosses the top chosen line on the monitor.
(v) Stop the ‘time meter’ by pressing the start/stop key again when the droplet crosses the bottom chosen line on the monitor
(vi) Stop the downward motion of the droplet by switching on the voltage.
(vii) Note the fall time from the ‘time meter’.
(viii) Press the ‘Clear key’ to make the ‘time meter’ read zero.
(ix) Allow the droplet to come below the bottom line by switching off the voltage.
(x) Switch on the voltage (its value already fixed earlier), the droplet will begin to rise.
(xi) Start the ‘time meter’ when the droplet crosses the chosen bottom line on the monitor.
(xii) Stop the “time meter’ when the droplet crosses the top chosen line on the monitor.
(xiii) Stop the upward motion of the droplet by switching off the voltage
(xiv) Note the rise time and the voltage.

Repeat these measurements of free- fall time and rise time several times. Take the average of these timings for the free-fall and the rise and note the voltage (kept fixed). Take another suitable droplet if it is there. If not, spray droplets again. Choose a suitable one and proceeded as above to get data on several droplets.

**b. <u>Balancing:</u>**

In the balancing method, the potential difference between the plates is adjusted to balance the droplets. Measure the free-fall time in the above 'Dynamic method' for the droplet to move between the two chosen lines on the monitor. Apply the voltage and adjust its value such that the electric force on the droplet just balances other forces on it and the droplet hangs (does not move up or down). The droplet should remain stationary for several minutes. The voltage may have to be adjusted again and again. Take the average value of these voltages and the average of the free-fall timings. Take another suitable droplet if it is there. If not, spray droplets again. Choose a suitable one and proceed as above to get on several droplets.

14. Note the atmospheric pressure and the room temperature at the end of the experiment as well.
15. Record your data in the format given below.
    This is slightly difficult experiment and needs some patience to achieve high accuracy. The more droplets one measures upon and the more data one takes on each droplet, the more accurate will be the final result.

    Note: Typical picture setting of TV are:
    (a) Brightness : 26
    (b) Contrast : 3
    (c) Color :0
    (d) Sharpness : 35

However, the above may be suitably adjusted to achieve maximum visibility of oil droplets.

**Observations:**

1. Distance d between the parallel plates = $5 \times 10^{-3}$ m

2. Distance L between the second line from top and fourth line (last but one line) on the monitor = 1 x 10 $^{-3}$ m
3. Density ‘ρ’ of oil = 929 kg/ m$^3$
4. Density ‘$\rho_a$’ of air = 1 kg/m$^3$
5. Room temperature T = 24 $^{O}$C
6. Atmospheric pressure P = mm of Hg =0.76 m
7. Coefficient of viscosity of air η from Fig. 3 = 1.8432 x 10$^{-5}$ kg/m.sec

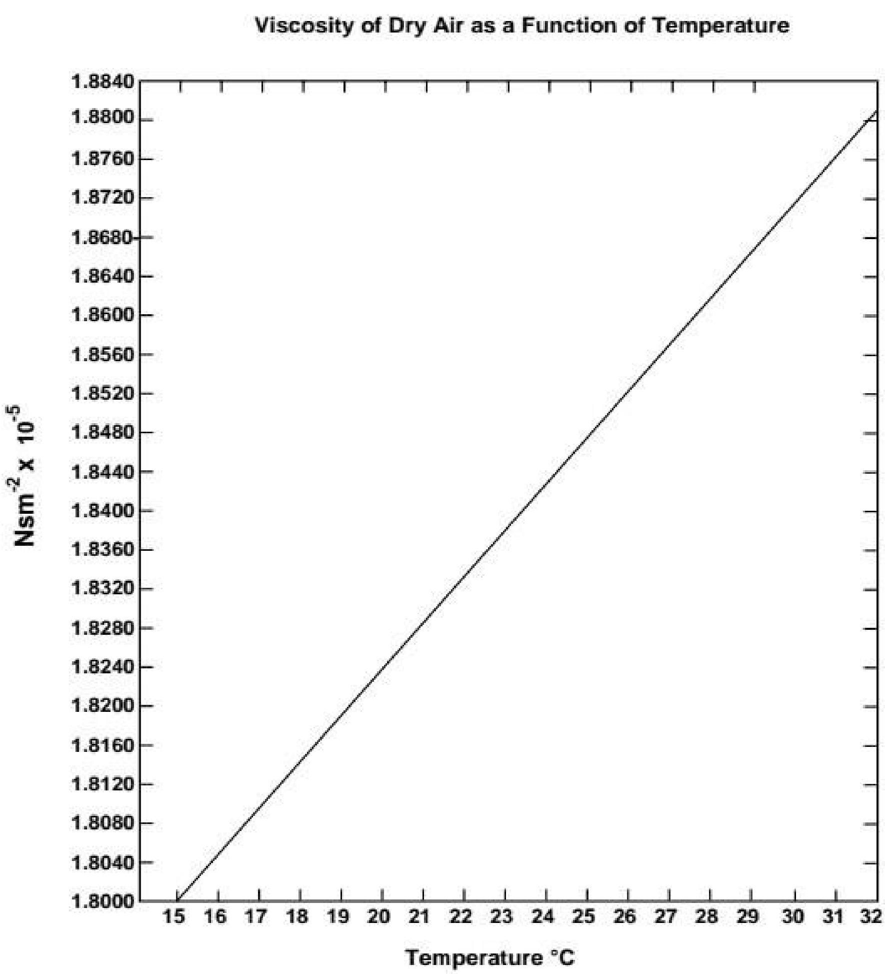


**Fig.3 Viscosity ‘η’ of dry air as a function of Temperature $^{O}$C.**

**Observational Data (Dynamic method):**

| Droplet No | S. No. | Free-fall Time | Rise Time | Mean Free-Fall Time ($t_f$) | Mean Rise Time ($t_r$) | Mean Free-fall Velocity ($v_f = L/t_f$ m/s) | Voltage (V volt) |
|---|---|---|---|---|---|---|---|
| 1 | 1 | | | | | | |
| | 2 | | | | | | |
| | 3 | | | | | | |
| | 4 | | | | | | |
| | 5 | | | | | | |
| 2 | 1 | | | | | | |
| | 2 | | | | | | |
| | 3 | | | | | | |
| | 4 | | | | | | |
| | 5 | | | | | | |
| 3 | 1 | | | | | | |
| | 2 | | | | | | |
| | 3 | | | | | | |
| | 4 | | | | | | |
| | 5 | | | | | | |
| 4 | 1 | | | | | | |
| | 2 | | | | | | |
| | 3 | | | | | | |
| | 4 | | | | | | |
| | 5 | | | | | | |
| 5 | 1 | | | | | | |
| | 2 | | | | | | |
| | 3 | | | | | | |
| | 4 | | | | | | |
| | 5 | | | | | | |
| 6 | 1 | | | | | | |
| | 2 | | | | | | |
| | 3 | | | | | | |
| | 4 | | | | | | |
| | 5 | | | | | | |

## Observational Data (Balancing Method)

| Droplet No. | S. No. | Free-fall Time | Mean Free-fall Time ($t_f$) | Mean Free-fall Velocity ($v_f = L/t_f$ m/s) | Balancing Voltage | Final Balancing Voltage ($V_b$ volt) |
|---|---|---|---|---|---|---|
| 2 | 1 | | | | | |
| | 2 | | | | | |
| | 3 | | | | | |
| | 4 | | | | | |
| | 5 | | | | | |
| 3 | 1 | | | | | |
| | 2 | | | | | |
| | 3 | | | | | |
| | 4 | | | | | |
| | 5 | | | | | |
| 4 | 1 | | | | | |
| | 2 | | | | | |
| | 3 | | | | | |
| | 4 | | | | | |
| | 5 | | | | | |
| 5 | 1 | | | | | |
| | 2 | | | | | |
| | 3 | | | | | |
| | 4 | | | | | |
| | 5 | | | | | |
| 6 | 1 | | | | | |
| | 2 | | | | | |
| | 3 | | | | | |
| | 4 | | | | | |
| | 5 | | | | | |

**Calculations:**

A. First calculate common constants to be used with all the droplets.

1. $C = 4\pi dg(\rho - \rho_a)/3 = 190.13$
2. $D = 9\eta/2g(\rho - \rho_a) = 9.04 \times 10^{-9}$
3. $\zeta = c/2P = 4.06 \times 10^{-8}$ where $c = 6.17 \times 10^{-8}$ m of Hg-m

B. **Calculation from the droplet data**

**Dynamic method**

| Droplet No. | $\xi$ (= $Dv_f$) | r (= $-\zeta + \sqrt{\zeta^2 + \xi}$) | $r^3$ | T (= $1 + \frac{t_f}{t_r}$) | ne ( = $CTr^3/V$) |
|---|---|---|---|---|---|
| 1 | | | | | |
| 2 | | | | | |
| 3 | | | | | |
| 4 | | | | | |
| 5 | | | | | |
| 6 | | | | | |

**Balancing Method**

| Droplet No. | $\xi$ (= $Dv_f$) | r (= $-\zeta + \sqrt{\zeta^2 + \xi}$) | $r^3$ | ne ( = $CTr^3/V_b$) |
|---|---|---|---|---|
| 1 | | | | |
| 2 | | | | |
| 3 | | | | |
| 4 | | | | |
| 5 | | | | |
| 6 | | | | |

For Balancing method voltage V is Final Balancing Voltage $V_b$.

### C. Analysis and treatment of the charge 'ne' on the droplets

1. Identify the minimum value of the charge ne on the droplets.
2. Divide the value of the charge ne on all the droplets by minimum value.
3. The result will be numbers close to integers for all the droplets.
4. Now extrapolates all these numbers or the earlier ones if no multiplication has been done to the nearest integer.
5. Divide the value of the charge ne on all the droplets by the respective integers.
6. This gives the values of the charge on an electron by different droplets
7. Take the average of these values. This gives the final result.

**Dynamic method Data**

| ne | ne divided by the lowest | nearest integer $n_{eff}$ | ne/ $n_{eff}$ |
|---|---|---|---|
| 1 | | | |
| 2 | | | |
| 3 | | | |
| 4 | | | |
| 5 | | | |
| 6 | | | |

**Balancing method Data**

| ne | ne divided by the lowest | nearest integer $n_{eff}$ | ne/ $n_{eff}$ |
|---|---|---|---|
| 1 | | | |
| 2 | | | |
| 3 | | | |
| 4 | | | |
| 5 | | | |
| 6 | | | |

**Comments:**

1. The oil provided with the set-up has (i) low evaporability so that the size of the drop does not change during the experiment and (ii) low variation in density with temperature.
2. The sprayer (atomizer) supplied with the equipment sprays droplets of suitable size with a relatively homogeneous cloud.
3. Try to setup the experiments on a levelled horizontal table. This will make the electrode plate horizontal so that the electric field between the plates is

vertical and is in line with the gravitational field. If there is a major shift in level, then the droplets will drift during the measurements and will go out of focus.

4. The measurements of the charge on the electron can lead to the calculation of Avogadro's number. The charge F (the Faraday) required to eletro-deposit one gram equivalent of an element on an electrode during electrolysis is equal to the charge of the electron multiplied by the number of molecules in a mole. The Faraday has been found to be $F = 9.625 \times 10^{7}$ coulombs per kilogram equivalent weight. Hence Avogadro's number $N = F/e$.

**Precautions:**

1. It is important to be spraying in the amount of oil sprayed into the chamber. It not only clogs the interior more quickly, but also creates so many particles in the field of the microscope that without excessive eye strain it is virtually impossible to single out and follow a single droplet.

2. The droplet should not be lost sight of during measurements. Working with these droplets is like a video game in which particle makes perfect.

3. The most important thing is to select appropriately sized droplets carrying very few elementary charges (say, 1e -3e). It is a good idea to make a preliminary analysis of the data for the first droplet immediately after it is obtained so that a judgement could be made as to which droplets to select and what voltages to use.

**References:**


1. R. A. Millikan, 'On the Elementary Electrical Charge and the Avogadro Constant'. Rev. 32 (1911) 349, a historical paper.
2. A. C. Melissios, 'Experiments in Modern Physics', Academic Press, N. Y. (1966).
3. S. Weinberg, 'The Discovery of Subatomic Particles' Scientific American Books, N. Y.(1883).
4. Michel Gagnon, Phys. Teach. 50, 98 (2012); https://doi.org/10.1119/1.3677285 (Also references in this article for theory)