Verification of Coulomb's law using Coulomb balance

Objectives:

(i) To study Coulomb's force as a function of the distance between two charges.
(ii) To study Coulomb's force as a function of charge.
(iii) To determine Coulomb's constant

Introduction:

The fundamental question in electrostatics is, given a set of charges located at certain positions, what would be the force on a given charge at a given position? The solution of this question is an empirical law that governs the forces (of repulsion or attraction) between two charges – the Coulomb's law. This electrostatic force or Coulomb's force is extremely strong in magnitude. For example, if one gram of protons and one gram of electrons are separated by 1 meter distance, the attractive Coulomb's force between them will be 1.5 x 10^{23} N. This is roughly the force needed to lift an object from the surface of the Earth with a mass about 1/5 that of the moon- not a small force. However, it is not so easy to measure such a force in the laboratory. Reason, the very magnitude of the force itself and secondly the tiny charge carriers as well as extremely mobile nature of the electrons. Therefore, while we can measure gravitational force by simply weighing an object, one needs a very delicate instrument such as a Coulomb balance to measure the much stronger Coulomb's force. Coulomb torsion balance occupies an extremely important place in the history of physics. Using this balance, Coulomb in 1785 developed a method for measuring the electrostatic force between two charged objects and confirmed that it depends on the charge and inverse square of the distance between two charged objects.

Theory:

If q_1 and q_2 are two point charges separated by distance R, then Coulomb's law is expressed as:

$$F = k_e \frac{q_1 q_2}{R^2} \qquad (1)$$

where k_e is the Coulomb constant. In SI units, the value of k_e is 8.9875 X 10^9 Nm^2/C^2. Coulomb's force acts along the direct line of separation between the two charges. Depending on the like or unlike nature of the point charges, the force is attractive or repulsive, respectively. Coulomb's force varies directly with the amount of charge and indirectly with R^2. A torsion balance gives a direct and reasonably accurate measurement of the Coulomb force. The validity of Coulomb's law has been subjected to intense scrutiny. The inverse square behaviour with the charge separation distances appears almost exact. One may write Coulomb force as, $F \propto 1/R^{2+\varepsilon}$, where ε is the deviation from the inverse square behaviour. Experimentally, one may fix limits on the maximum magnitude of ε, depending on the sensitivity and accuracy of the experiment. Value of ε determined by Cavendish and Maxwell in earlier times were $< 10^{-2}$ and $< 10^{-6}$, respectively. With modern experiments this has improved to $< 10^{-16}$. Today, Coulomb force is believed to obey inverse square behaviour exactly. Another amazing aspect of the Coulomb law is the range of length scales where it has been tested and found valid. Coulomb's law is confirmed down to length scales of 10^{-15} m while measurements on the magnetic field of Jupiter have confirmed this law to the large length scales of 10^{8} m.

Coulomb's torsion balance:

The historical torsion balance designed by Coulomb is shown in Fig.1. It comprised of a cylindrical glass case and closed by a lid from which a glass tube emerges out. The tube ends with a piece of metal from which a torsion fibre is suspended. This fibre holds a horizontal needle made of lac, with a brass disc A at one end and a sphere at the other. The height of the needle is adjusted by a knob which is turned to wind the suspending thread on a horizontal axis. This axis is mounted on a revolving disc on which is engraved a scale calibrated in degrees. The second sphere B is suspended through a hole on the lid of the glass case. B is charged outside the case and placed back touching the brass disc and thereby charging it too. The two charged objects were found to repel one another, twisting the fibre through a certain angle, which could be read from a scale on the instrument. By knowing the angle, Coulomb was able to calculate the force between the balls.

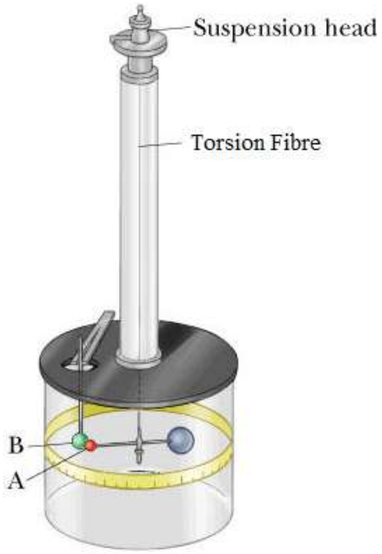

Fig. 1

Faraday's ice pail:

To demonstrate the effect of electrostatic induction on a conducting container, Faraday used a metal pail meant for holding ice. The experiment shows that an electric charge enclosed inside a conducting shell induces an equal charge on the outside of it. It also demonstrates the principles behind electromagnetic shielding normally used in the *Faraday cage*. The ice pail experiment was the first precise *quantitative* experiment on electrostatic charge. Charge carried by an object can be measured using an electrometer by placing the object in a grounded ice pail.

Unit of charge:

Because of the difficulty of making accurate electrostatic measurements directly, the SI unit of charge, the *coulomb* (C), is defined in terms of the unit of electric current, the *ampere* (A). If a steady current of one ampere is flowing through a wire, then one coulomb is the amount of charge passing through any cross section of that wire in one second, or 1 A = 1 C/s. The charge on a single electron is 1.602×10^{-19} C.

Description of the set up:

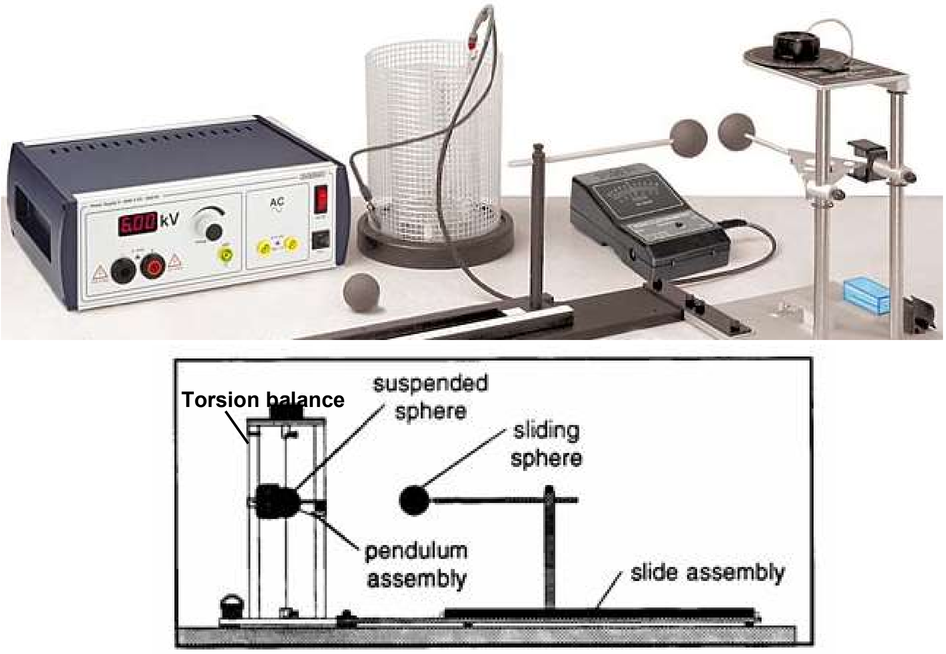

Fig. 2: Compete experimental set up and its schematics

The complete set up used in this experiment is shown in Figure 2. The PASCO Model ES-9070 Coulomb Balance is a delicate torsion balance that can be used to investigate the force between charged objects. A conductive sphere with radii of 1.9 cm is mounted on a rod, counterbalanced, and suspended horizontally from a thin torsion wire. The torsion wire is connected to the torsion knob, passing through the counterweight vane, and connected to the torsion wire retainer. The torsion knob is attached to a degree scale, which indicated how many degrees the knob was turned. Another identical sphere is mounted on a slide assembly so that it can be positioned at various distances from the suspended sphere.

To perform the experiment, both spheres are charged, and the sphere on the slide assembly is placed at fixed distances from the equilibrium position of the suspended sphere. The electrostatic force between the spheres causes the torsion wire to twist. The angle through which the torsion wire must be twisted to re-establish equilibrium is directly proportional to the electrostatic force between the spheres. One can verify the inverse square relationship and the charge dependence using the balance and any electrostatic charging source. A stable kilovolt power supply (0-6.6 kV) is used to charge the spheres. An electrometer and a Faraday ice pail are provided for accurate measurement of the charge on the spheres. For more detail information about the set up and initial adjustment, please refer the company manual.

Corrections to the data

The inverse square relationship has been found to deviate at short distances due to the fact that the charged spheres are not simply point charges. A charged conductive sphere, if it is isolated from other electrostatic influences, acts as a point charge. The charges distribute themselves evenly on the surface of the sphere, so that the centre of the charge distribution is just at the centre of the sphere. However, when two charged spheres are separated by a distance that is not large compared to the size of the spheres, the charges will redistribute themselves on the spheres so as to minimize the electrostatic energy. The force between the spheres will therefore be less than it would be if the charged spheres were actual point charges.

A correction factor can be used to account for this deviation. Using method of image charge, a first order correction, B, can be calculated as

$$B = 1 - 4\frac{a^3}{R^3} \qquad (2)$$

where a equals the radius of the spheres and R is the separation between spheres. Thus the corrected Coulomb force can be written as

$$F_{corr} = B.k_e\frac{q_1q_2}{R^2} \qquad (3)$$

Torsion constant of the fibre:

When the torsion fibre is twisted by an angle θ, the resulting torque is proportional to θ. In the present set up since the torque arm is always the same, the torsion force becomes proportional to θ. Thus

$$F_{tor} = K_{tor}.\theta \qquad (4)$$

where K_{tor} is the proportionality constant is termed as torsion constant K_{tor}. The value K_{tor} can be determined by measuring torsion force as a function of θ and calculating K_{tor} from the slope of a graph plotted between $F_{tor} \sim \theta$. Details about this measurement is provided in the company manual. The value of K_{tor} is pre-determined following the suggested method and hence you don't have perform this part during your experiment. At equilibrium Coulomb force F_{corr} is balanced by F_{tor}. Hence using Eqns (3) and (4), a general working formula can be derived showing the relation between the angle of twist, charge on the spheres and the separation distance between them:

$$\boxed{\boldsymbol{\vartheta_{corr} = \frac{k_e}{K_{torr}} \frac{q_1 q_2}{R^2}}} \qquad (5)$$

where $\vartheta_{corr} = \vartheta / B$ and K_{torr} = ...

Operating Tips:

This experiment works best in winter when the air is dry and charge will not leak rapidly from the spheres. Keep the balance away from the walls or people which might be charged. Stand behind and away from the balance, and touch ground to lose any charge on yourself. After charging the spheres, turn off the power supply immediately. Keep your hands as far as possible from the sphere while charging it. If the charge seems to be leaking away rapidly, clean the insulators with alcohol. Note that the high voltage supply is turned all the way up to around 6kV. Do not touch the end of the high voltage probe or else you will receive a moderate shock. The shock is moderate because the high voltage supply has a very large resistor in series with the probe, limiting the current flow to a safe level.

Procedure:

(A) Force as a function of distance:

1. Go through the Precautions/Tips list first which is provided at the end of the manual and follow it strictly for better results.
2. Before beginning the experiment make sure that the spheres are fully discharged (touch them with a grounded probe) and move the sliding sphere as far as possible from the suspended sphere.
3. The coulomb balance is pre-adjusted and aligned and usually ready for the experiment. Just ensure that the torsion dial is at 0^0. In case not, do some fine adjustment by appropriately rotating the bottom torsion wire retainer (marked in Fig. 3) until the

pendulum assembly is at its zero displacement position as indicated by the index marks.

4. Switch on the high voltage power supply to set the voltage output at 6-7 kV and then switch off. One terminal of the power supply should be grounded.
5. With the spheres still at maximum separation, charge both the spheres to a potential of 6-7 kV, by touching them one by one with a charging probe. Immediately after charging the spheres, turn the power supply off to avoid high voltage leakage effects.

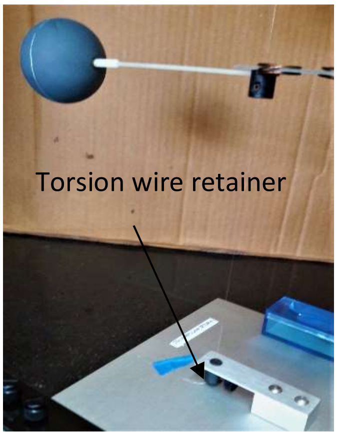

Fig. 3: Zero displacement adjustment

6. Position the sliding sphere at a position of about 20 cm. Due to same charge on both spheres, the suspended sphere moves away from the sliding sphere to an equilibrium position. Adjust the torsion knob as necessary to balance the forces and bring the pendulum back to the zero position. Repeat this measurement several times, until your result is repeatable to within $\pm 1^0$. Do not forget to discharge the spheres by touching them with a grounded probe, every time you take a measurement. Record the distance (R) and the angle (θ) in Table 1.
7. Separate the spheres to their maximum separation and make sure that they are fully discharged. Keeping the supply voltage same, repeat steps 4-5 by positioning the sliding sphere at different separation distances in the range 6-20 cm in steps of 1 cm. This range works best when the humidity range is 29-35%. If humidity is more or less vary the range accordingly to get best results.
8. To measure charge on the spheres (both the spheres acquire same charge), you need to use a third conducting sphere identical to the other two. The third sphere is attached to an insulating thread in your set up. Make sure that it is discharged by touching it to a grounded probe.
9. You will also need to know the effective capacitance C of the electrometer, ice pail and the connecting cords when they are connected. Use an LCR meter once to record this value.

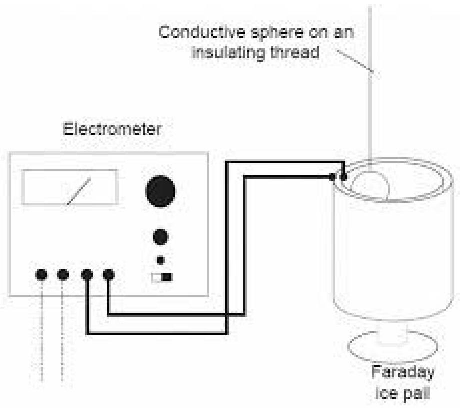

Fig. 4: Measurement of charge

10. Now, using the desired potential from the power supply, charge the sliding sphere. Hold the third sphere by the insulating thread, touch it to the charged sphere first and then to the

inner conductor of the Faraday ice pail as shown in Fig. 4. Since the capacitance of the ice pail and electrometer is much greater than that of the sphere (you can verify this), virtually all of the charge q is transfer on to the ice pail.

11. Note down the voltage reading (V) of electrometer. The charge on the sliding sphere q, can now be determined using the equation q = 2CV. The factor 2 arises because the third sphere acquires only half of the actual charge on the sliding sphere.

(B) **Force as a function of charge:**

1. In this part, keeping the separation distance constant, the spheres are charged to different values and the corresponding torsion angles are noted.
2. Make sure again that the spheres are fully discharged (touch them with a grounded probe) and move the sliding sphere as far as possible from the suspended sphere.
3. Ensure that the set up is balanced with the torsion dial at 0^0.
4. Set the power supply at 2 kV and switch off. Charge the two spheres separately by switching on the power supply now. Thus both spheres are ideally charged to the same value.
5. Quickly move the sliding sphere to a particular separation distance (choose a value between 8 and 10 cm). The suspended sphere is deflected away from the equilibrium position. Note the angle rotated to bring the balance back to equilibrium.
6. Keeping R constant, repeat steps 4-5 each time charging the spheres to different values by setting the power supply at various voltages in the range 2 – 6kV.
7. Follow steps 9 and 10 from section A to measure the charge on the spheres for each supply voltage.

Observations:

K_{torr} = ... C = ... B = ...

Table -1: Supply voltage = ... Charge on each sphere = ...

Sl#	R	ϑ	ϑ_{corr}

Table – 2: R = ...

Sl#	Supply voltage	Charge	ϑ	ϑ_{corr}

Graphs and analysis:

1. Using data from section A plot ϑ_{corr} ~ R and fit an appropriate function to check inverse square relation. Justify the deviations in your data.
2. Plot ϑ_{corr} ~ $(1/R^2)$, fit the data with a straight line.
3. Plot ϑ_{corr} ~ (q^2), fit the data with a straight line.

(C) Determination of Coulomb's constant:

Determine Coulomb's constant from the slopes of graphs 2 and 3.

Precautions/Tips for accurate result:

1. If you live in an area where humidity is always high, and if you have no facilities for controlling humidity, the experiment will be difficult, if not impossible, to perform. Static charges are very hard to maintain in a humid atmosphere because of surface conductivity.
2. As with any quantitative electrostatic experiment, things like a charged shirt sleeve, an open window, an excessively humid day etc can affect your experiment. However, if you carefully follow the tips listed below, you have got a good start toward a successful experiment.
3. Position the torsion balance at least two feet away from walls or other objects which could be charged or have a charge induced on them. When performing experiments, stand directly behind the balance and at a maximum comfortable distance from it. This will minimize the effects of static charges that may collect on clothing.
4. Avoid wearing synthetic fabrics, because they tend to acquire large static charges. Short sleeve cotton clothes are best, and a grounding wire connected to the experimenter is helpful.

5. When charging the spheres, turn the power supply on, charge the spheres to the desired value and then immediately turn the supply off. The high voltage at the terminals of the supply can cause leakage currents which will affect the torsion balance.

6. When charging the spheres, hold the charging probe near the end of the handle, so your hand is as far from the sphere as possible. If your hand is too close to the sphere, it will have a capacitive effect, increasing the charge on the sphere for a given voltage. This effect should be minimized so the charge on the spheres can be accurately reproduced when recharging during the experiment.

7. Surface contamination on the rods that support the charged spheres can cause charge leakage. To prevent this, avoid handling these parts as much as possible and occasionally wipe them with alcohol to remove contamination.

8. There will always be some charge leakage. Perform measurements as quickly as possible after charging, to minimize the leakage effects.

9. Discharge the spheres completely and recharge them before each measurement. Remember that the spheres should be at maximum separation while charging them.

Ref:

1. Company manual
2. http://pms.iitk.ernet.in/wiki/index.php/Coulom’s_law_its_experimental_verification_and_validity