Numerical Computation
NISER-ML Semester 4 of 2009

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1. **Error in Computation [2, 3]**
   (a) Floating point representation and arithmetic;
      i. Normalised mantissa;
      ii. Overflow, underflow;
      iii. round-off error ($fl(x) = x(1 + \delta)$);
      iv. Rounding: $fl(x)$ nearest floating point number to x, using floating point number whose last stored digit is even in case of tie;
      Chopping: Truncate base-β expansion of x after $(n-1)$st digit;
      bound of δ: $|\delta| < \frac{1}{2}\beta^{1-n}$ in rounding, $-\beta^{1-n} < \delta \leq 0$;

   (b) Error = Exact - Approximation, absolute error, relative error;

   (c) Loss of significant digits: If x^* is an approximation to x, then we say that x^* approximates x to r significant β-digits provided the absolute error is at most $\frac{1}{2}$ in the rth significant β-digit of x.

   $$|x - x^*| \leq \frac{1}{2}\beta^{s-r+1}$$

   where s is the largest integer such that $\beta^s \leq |x|$.

   (d) **Assignment 1:** Find the roots of the quadratic equation $x^2 + 111.11x + 1.2121$ upto 10 significant digits.

   (e) Approximating by an infinite series.

   (f) $\frac{\pi}{4} = \sum_{i=0}^{\infty} \frac{-1}{2i+1} = 1 - \sum_{j=1}^{\infty} \frac{2}{16j^2-1}$ [2]. Follow α_n according [2, Example, Page 20] to compute n such that $\pi/4$ is correct upto 10^{-6}.

   (g) Truncation error : The error caused by terminating after a finite number of terms is called trunction error.

   (h) Taylor's Expansion.

   (i) **Assignment 2:** Compute $e, \sqrt{e}, \log 2, \frac{1}{\pi}(using\ \frac{sinx}{x})$ correct upto 10 significant 10-digit.

2. Appoximating the zeros of non-linear equations [1, 2]
   (a) Bisection method;
   (b) Newton's method;
   (c) Secant method;
   (d) Analysis of their concergence;
   (e) **Assignment 3:** Compute a zero of $e^{2x} - e^x - 2, x^6 - x - 1, e^x - 3x^2, x + 0.3cosx + 1, x - 0.2sinx - 0.5$ correct upto 10 significant 10-digit using above 3 methods and the number of steps require for each method and see their rate of convergence.

3. Interpolation [1]

(a) Uniqueness of interpolating polynomial of $n+1$ points by n-degree polynomial;

(b) Langrange interpoltaion;

(c) Newton divided differences.

(d) **Assignment 4:** Interpolate e^x, $\frac{1}{x+3}$, $\log(1+x)$ at $x = p$ for integers $0 \le p \le 4$ using Langrange interpolation method and Newton divided difference method. Tabulate the error of the polynomial at $0.25t$ for $0 \le t \le 20$ upto 10 decimal places. Repeat the above by interpolating at $x = 0.5p$ for integers $0 \le p \le 8$.

4. Numerical Integration [1]

   (a) Integration using approximation of function and series expansion of function;

   (b) $E_n(f) = I(f) - I_n(f) = \int_a^b [f(x) - f_n(x)]dx$
   $|E_n(f)| = |I(f) - I_n(f)| = \int_a^b |f(x) - f_n(x)|dx \le (b-a)\|f - f_n\|_\infty$

   (c) Trapezoidal rule, Error $(-\frac{(b-a)^3}{12} f''(\eta), \eta \in [a,b])$ trapezoidal rule on sub interval;

   (d) Simpson's rule;

   (e) Newton-Cotes rule;

   (f) **Assignment 5:** Calculate $\int_0^1 e^{-x^2} dx$, $\int_0^{2\pi} \frac{1}{2+cos(x)}$, $\int_0^{\pi} e^x cos(4x)dx$ using the trapizoidal rule, simpsons rule and newton-Cotes rule(with 4 an 5 points) with n with $n = 2, 4, 8, \ldots, 1024$ and analyse the error of convergence for each rules calculating the ratios of
   $$R_n = \frac{I_{2n} - I_n}{I_{4n} - I_{2n}}$$

5. Solving ODE [1, 2]

   (a) Euler's method(using Taylor's series);

   (b) Midpoint method;

   (c) Runge-Kutta method of order 2 and order 4 [2];

   (d) Adams-Bashforth multi-step method [2];

   (e) **Assignment 6:** For the equation $y' = -2y, 0 \le x \le 1, y(0) = 1$, compute $y(x)$ on the step size of .25, .05, .1 and .2 using the above five methods and write a conclusion of the error of convergence. (Follow the table and analysis in the book by Atkinson at the page 344)

   (f) Solving boundary value problem using finite difference method.

   (g) **Assignment 7:** Exercises 9.1.1, 9.1.2 [2]

6. Continuing ...

References

[1] K. E. Atkinson. *An Introduction to Numerical Analysis*. Willey, 2004.

[2] S. D. Conte and C. D. Boor. *Elementary Numerical Analysis*. Tata McGraw-Hill, 2006.

[3] R. J. Schilling and S. L. Harries. *Applied Numerical Methods for Engineers*. Thomson Brooks/Cole, 2006.

$Name$	$Assignments$	$Total$
$Ashish$	$A1, A3$	
$Sneha$	$A1, A3$	